

# **SIMPLE MODEL FOR THE ENERGY SUPPLY OF A STAND-ALONE HOUSE USING HYBRID WIND–SOLAR POWER SYSTEM**

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## **ABSTRACT**

*A research project for secondary school students involving both physical measurements and modelling is presented. The problem to be solved is whether and how a typical house can be supplied with energy off-grid, based entirely on renewable energy sources, more specifically, on solar and wind energy, while using relatively simple devices, namely, photovoltaic modules, wind turbines and accumulators. Our students carried out a long term measurement series in order to assess typical energy consumption of houses. Further, the number of solar modules and wind turbines and the necessary accumulator capacity was estimated.*

## **INTRODUCTION**

Renewable energy sources are becoming increasingly important in energy supply. Their contribution covered an estimated 19% of the global energy consumption in 2011 [1]. The application of the renewable energy resources helps to reduce global greenhouse gas emission and mitigate global warming [1, 2, 3]. Due to their significance and perspective, it is desirable to give renewable energy sources an appropriate share in physics teaching. In this paper a related research project designed for and accomplished by secondary school students is described. Our students examined the role of the renewable energy sources and the behaviour of an off-grid hybrid wind-solar model system using elementary concepts of physics. In this model electricity was generated by means of photovoltaic (PV) modules and wind turbines, while the electrical energy was stored in accumulators. A quite similar model was discussed in [4].

We can make the traditional lessons more colourful with different project tasks. The well-planned project tasks augment students' knowledge in the particular topic and make students more motivated in learning that subject. According to my experience the most effective tasks were carried out in pairs or groups with the teacher's guidance, as well as in this project.

Our 'Renewable energy sources: stand-alone house with hybrid wind-solar power generator' project has been carried out in three stages. For the first stage the daily energy consumption of an average house was investigated. Energy consumption of electric home appliances and energy consumption of air conditioner and heating energy were monitored separately. During the project, which lasted for 2 years (from 2012-Oct-1 to 2014-Oct-1), the students measured the daily energy consumption of their households. The wind speed and the sunlight were also monitored. For the second stage we developed a mathematical model for an off-grid house with hybrid wind–solar power generator and accumulator system. For the third stage a computer simulation program was developed, based on the mathematical model and the data collected by students. This programme enabled the simulation of the energy system of an off-grid house. The feasibility of the model was also analyzed; in particular the necessary accumulator capacity was determined. The main purpose of the project was, however, educational.

## MODELLING

The model setup is depicted schematically in Fig.1. The parts of the system are the power generating system (photovoltaic modules and wind turbines), the energy storage unit (accumulator system); and the appliances: electric home appliances, electric heating system and cooling system. The solar modules are mounted on the roof.

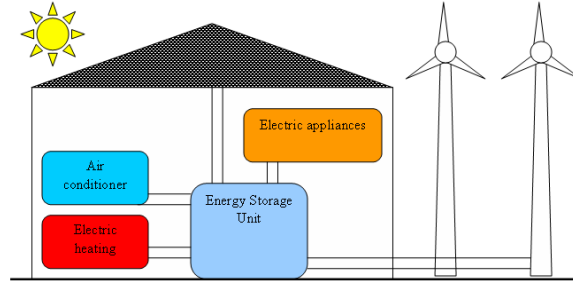


Fig.1. The stand-alone model house with hybrid wind–solar system and energy storage unit

### Gathering data

Our 16 - 18 old year students took part in this project voluntarily. The number of students taking part in the project was  $N=31$ . Measurements were carried out partly at home and partly in extracurricular afternoon physics classes in team work. Students collected the data of the daily energy consumption of their own houses: the energy consumption of the electric appliances was monitored; the natural gas consumption was monitored by gas meter; the wood and coal burned in furnaces were measured in weighing-machines (scales). The electrical energy consumption of the air conditioner was monitored (estimated) separately from the electric home appliances. The data was gathered by students then we counted the averaged values on a daily basis:

$$E_{total,i} = \frac{A_{ave} \cdot N_{ave}}{N} \sum_{j=1}^N \frac{E_{total,i,j}}{A_j \cdot N_j}, \quad (1)$$

where  $E_{total,i}$  is total averaged energy consumption of the model house on  $i$ th day,  $A_{ave}$  is average floor-space of houses,  $N_{ave}$  is the average number of inhabitants in the house,  $A_j$  is floor-space of the house of  $j$ th student,  $N_j$  is the number of inhabitants in the house of  $j$ th student,  $E_{total,i,j}$  is total energy consumption of the house of  $j$ th student on  $i$ th day.

In the given 2-year period the average daily electricity consumption per household was 36.08 MJ; the lowest average daily electrical energy consumption was 27.44 MJ and the highest one was 41.89 MJ. (Before doing the project, most of the students could not even estimate the order of the energy consumption of their own home. During the project they learnt to collect, process and analyze data in the long run.)

We monitored the local outside temperature as well as the inside temperature during the project. The wind speed, the air pressure and the sunlight were monitored in every  $\Delta t=5$  minutes automatically by the local weather station, so these data were available for us.

### PV modules

A photovoltaic (PV or solar) cell converts the energy of light directly into electricity by the photovoltaic effect. A photovoltaic module is built from blocks of photovoltaic cells. The power of a photovoltaic module is proportional to the incoming light power [4]. The total energy production of the photovoltaic modules on  $i$ th day can be calculated as

$$E_{photov,i} = \sum \eta_{photov} \cdot A_{photov} \cdot I \cdot N_{photov} \cdot \Delta t, \quad (2)$$

where  $A_{photov}$  is the area of one PV module,  $I$  is the light intensity (incident solar flux in  $\text{W}/\text{m}^2$ ),  $\eta_{photov}$  is the efficiency of the solar module on  $i$ th day and  $N_{photov}$  is the number of PV modules.

### Wind turbines

Wind turbine generates electricity from the kinetic energy of the wind. In our model small (not commercially serialized) wind turbines are used to generate electricity. For simplicity the turbines are assumed to be self-orientating devices. The power output of the wind turbine is proportional to the cross sectional area swept by the rotor and to the cube of the wind velocity [5, 6]. The total energy production of the wind turbines on  $i$ th day can be calculated as

$$E_{windt,i} = \sum C_{po} \cdot \frac{\rho_{air}}{2} \cdot A_{rotor} \cdot v_{wind}^3 \cdot N_{windt} \cdot \Delta t, \quad (3)$$

where  $\rho_{air}$  is the density of air,  $A_{rotor}$  is the cross sectional area of rotor,  $v_{wind}$  is wind velocity,  $C_{po}$  is the power coefficient, and  $N_{windt}$  is the number of wind turbines. The density of air is  $\rho_{air}=p \cdot M \cdot R^{-1} \cdot T^{-1}$ ; where  $p$  is the pressure (monitored),  $T$  is the absolute temperature of air (monitored),  $M=0.029 \text{ kg} \cdot \text{mol}^{-1}$  is the molar mass, and  $R$  is the universal (molar) gas constant.

### Accumulators

The electricity demand of the model house can change significantly on a smaller timescale. When electrical energy is generated in solar modules and/or in wind turbines, it gets stored instantly in accumulators according to the model assumption. We discussed what size of accumulator capacity ( $E_{acc\_max}$ ) is suitable for the parameters given in our off-grid system. We must take a battery system large enough to prevent blackouts (total energy loss) in the whole period of the project.

## ENERGY INPUT AND OUTPUT AND ENERGY STORAGE

In order to determine the necessary storage capacity of batteries, we studied the energy inputs (produced energy) and outputs (dissipated energy) of the system in detail.

### Heat transmission

Temperature difference in any situation results in energy flow into the system or energy flow from the system to its surroundings. The former leads to the heating, the latter leads to the cooling of the system. The total energy flow of heat transmission process on  $i$ th day is

$$E_{heatr,i} = \sum U \cdot A_f \cdot (T_{out} - T_{in}) \cdot \Delta t, \quad (4)$$

where  $A_f$  is the free surface area of building,  $U$  is the overall heat transmission coefficient,  $T_{out}$  is the absolute temperature of ambient air,  $T_{in}$  is the absolute inner temperature of the building.

### Thermal radiation

The accurate analysis of heat radiation of the system is a complex problem; so we try to construct only an approximate model accounting for thermal radiation. The total energy flow of heat radiation process on  $i$ th day:

$$E_{rad,i} = \sum \varepsilon \cdot \sigma \cdot A_f \cdot (T_{out}^4 - T_{in}^4) \cdot \Delta t, \quad (5)$$

where  $\sigma$  is Stefan-Boltzmann constant ( $\sigma=5.67 \cdot 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$ ),  $\varepsilon$  is the overall (average) emissivity of the building.

### Heating

Our building is an off-grid system and has electric heating; that is, electric current through a resistor releases heat. The total electrical energy consumption of resistance heating on  $i$ th day:

$$E_{heating,i} = \sum P_{heating} \cdot \Delta t, \quad (6)$$

where  $P_{heating}$  is the electric power of resistance heater.

In the model if the required inside temperature of the building is higher than the actual inner temperature, then we use the electric heating system. If the instantaneous outside temperature is

higher than the required inside temperature of the building in the ‘heating season’ then it is not necessary to use the heaters, we can warm the interior of the house by opening the windows.

### Cooling

If the temperature inside the house is too high (in summer) we can use an air conditioner; it is a device that lowers the air temperature. The cooling process is typically achieved through refrigeration cycles. The total electrical energy consumption of air conditioner on  $i$ th day:

$$E_{cooling,i} = \sum P_{aircond} \cdot \Delta t, \quad (7)$$

where  $P_{aircond}$  is the electric power of the air conditioner.

In the model if the required inside temperature of the building is lower than the actual inner temperature, then we use the air conditioner. If the instantaneous outside temperature is lower than the required inside temperature of the building in the ‘non heating season’, then it is not necessary to use the air conditioner, we can cool the house interior by opening the windows.

### Internal energy

Our building can store energy as internal energy. The internal energy of a macroscopic system at a given temperature is proportional to its heat capacity [7]. The internal energy of model house on  $i$ th day is assumed to be

$$E_{internal,i} = C_{air} \cdot T_{in,i} + C_{wall} \cdot \frac{T_{in,i} + T_{out,i}}{2}, \quad (8)$$

where  $C_{wall}$  is the heat capacity of walls, and  $C_{air}$  is the heat capacity of air inside the building.

## ENERGY BALANCE

Now the energy balance of the hybrid wind-solar power generating system is considered.

### Heating season

It is supposed for the sake of simplicity that in the ‘heating season’ the internal energy of the model house on  $i$ th day ( $E_{internal,i}$ ) depends on the internal energy on the previous day ( $E_{internal,i-1}$ ), the net electric heating on  $i$ th day (if any), and the net energy flowing in or out of the system by heat transfer ( $E_{heattr,i}$ ) and heat radiation process ( $E_{rad,i}$ ) on  $i$ th day:

$$E_{internal,i} = E_{internal,i-1} + \eta_{heating} \cdot E_{heating,i} + E_{heattr,i} + E_{rad,i}, \quad (9)$$

where  $\eta_{heating}$  is the efficiency of the electric heating device. In this model resistance heater is applied, thus  $\eta_{heating}$  is nearly 1 because an off-grid electric resistance heater converts (nearly) the full electric energy into heat. (Some energy is needed by ventilators, if any.)

According to the household’s need, the total daily electrical energy consumption on  $i$ th day is the energy of electric home appliances and electric heating. In the ‘heating season’ the energy stored in accumulators at the end of  $i$ th day ( $E_{acc,i}$ ) depends on the energy stored in batteries at the end of the previous day ( $E_{acc,i-1}$ ), the total electrical energy produced by PV modules ( $E_{photov,i}$ ) and wind turbines ( $E_{windt,i}$ ) on  $i$ th day, and the total energy dissipated in the electric resistance heater ( $E_{heating,i}$ ) and electric home appliances ( $E_{eapp,i}$ ) on  $i$ th day:

$$E_{acc,i} = E_{acc,i-1} + E_{photov,i} + E_{windt,i} - E_{heating,i} - E_{eapp,i}. \quad (10)$$

### Non-heating season

In the ‘non-heating season’ the internal energy of model house on  $i$ th day ( $E_{internal,i}$ ) depends on the internal energy of the house on the previous day ( $E_{internal,i-1}$ ), the net cooling energy on  $i$ th day (if any), and the net energy flowing in or out the house by heat transfer ( $E_{heattr,i}$ ) and heat radiation process ( $E_{rad,i}$ ) on  $i$ th day:

$$E_{internal,i} = E_{internal,i-1} - C_{cooling} \cdot E_{cooling,i} + E_{heattr,i} + E_{rad,i}, \quad (11)$$

where  $C_{cooling}$  is the coefficient of performance (COP) of the air conditioner.

According to the household's need the total daily electrical energy consumption on  $i$ th day is the energy of electric home appliances and air conditioning. In the 'non-heating season' the energy stored in accumulators at the end of  $i$ th day ( $E_{acc,i}$ ) depends on the energy stored in batteries at the end of the previous day ( $E_{acc,i-1}$ ), the total electrical energy produced by PV modules ( $E_{photov,i}$ ) and wind turbines ( $E_{windt,i}$ ) on  $i$ th day, and the total energy dissipated in air conditioner ( $E_{cooling,i}$ ) and electric home appliances ( $E_{eapp,i}$ ) on  $i$ th day:

$$E_{acc,i} = E_{acc,i-1} + E_{photov,i} + E_{windt,i} - E_{cooling,i} - E_{eapp,i}. \quad (12)$$

## SIMULATIONS

We used spreadsheet software for the simulations. This method enables the solution of a complex physical (mathematical) problem in a relatively simple way. In the simulations we 'estimated' the energy consumption of a typical house with 4 inhabitants. We tried to choose realistic data for the simulations according to the data gathered by students. Our model house is a hollow rectangular building, with  $A_{ave}=100 \text{ m}^2$ ; the dimensions are  $a=10\text{m}$ ,  $b=10\text{m}$ ,  $h=3\text{m}$  (height), the thickness of walls is  $d=0.4 \text{ m}$ . In Fig. 2. the electrical energy produced by photovoltaic modules and wind turbines can be seen in the 2-year period of project (from 2012-Oct-1 to 2014-Oct-1). The power coefficient of the not commercially serialized wind turbine is  $C_{po}=0.25$ , the cross sectional area of rotor is  $A_{rotor}=4 \text{ m}^2$ , the number of wind turbines is  $N_{windt}=4$ . The area of one PV module is  $A_{photov}=1 \text{ m}^2$ , the number of solar modules is  $N_{photov}=100$ , the efficiency of the solar module is  $\eta_{photov}=0.15$ .

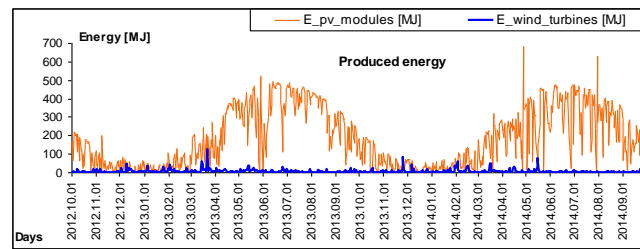


Fig.2. Electrical energy produced by PV modules and wind turbines during the whole period of the project

In Fig.3. the electrical energy consumption of the model house (electrical home appliances, electric heater and air conditioner) is shown in the 2-year period of the project (from 2012-Oct-1 to 2014-Oct-1). The efficiency of resistance heating device is  $\eta_{heating}=1$ , the coefficient of performance of the air conditioner is  $C_{cooling}=3$ . The overall heat transmission coefficient of insulated walls is  $U=0.18 \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-1}$ ; the emissivity of the building's walls is  $\varepsilon=0.12$ .

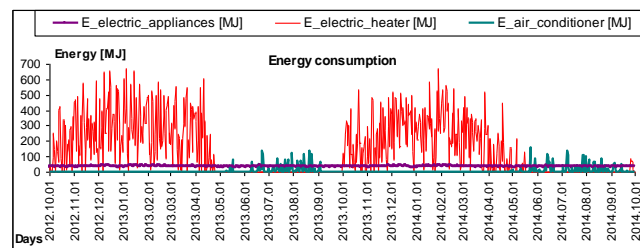


Fig.3. Electrical energy consumption of the house (electric home appliances, electric heater and air conditioner) during the whole period of the project

Computer simulations with spreadsheet software were performed in order to determine the necessary capacity of the storage unit [8]. In the simulation with the given data the necessary capacity of the energy storage unit that must be chosen is approx. 45097 MJ in order to prevent blackouts (in order to  $E_{acc,i}$  take values only between 0 and  $E_{acc,max}$ ) every day in the 2-year period of the project. We got that the capacity of the accumulator system derived from the simulations has a value too large for a real-world storage system. It cannot be realised in the real world in a house.

### Without electric heating

Electric heating is the biggest form of energy consumption in the model house. If the electric heating is rejected and fossil fuel (e.g. wood) heating is applied, then the necessary capacity of storage unit is approx. 547.5 MJ according to our simulation. In this case all electric home appliances and even the air conditioner can be used in the model house in the whole period of the project [8]. This storage capacity might be realised (perhaps), but it would be very expensive.

### CONCLUSIONS

Students had to consider some properties of a stand-alone house with a hybrid wind–solar power generator and accumulator system. The parameters of the model house and the dimensions of the hybrid wind-solar power generator system are fitted to data collected by students. Discrete energy balance equations were given to determine the necessary capacity of the energy storage unit. We think that this student project helps to strengthen connection between theory and practice, improving practice within the field of physics education. We hope that this simplified model can be profitable for interested students in grammar schools.

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