THE SLEDGE PROJECT

C. Fülöp[1,2], R. Szabó[1],T. Berényi[1], B. Simó[1]
[1]Trefort Ágoston Bilingual Technical High School, Budapest, Hungary, fulcsilla@gmail.com
[2]Physics Education PhD Program, Eötvös University, Budapest

ABSTRACT

"Why is it easier to pull a sledge horizontally than up a slope?" It is a well-known question often asked by physics teachers. This is an issue in the classroom when we analyze friction and motion on a slope. The answer is simple; at least it seems to be. Our little group made a deeper study of the question. We did a Newtonian analysis by giving the equations of motion. This led us to a function of two variables. The analysis of this function goes beyond the secondary school curriculum. We numerically analyzed it with a program written in C++. We measured typical tilt angles of sledging hills and typical friction coefficient values of snow-sledge surfaces. "It is easier to pull something on level ground than up a slope." Is this statement generally or exclusively true in the case of specific conditions?

INTRODUCTION

While travelling home from a physics competition with eager students, we discussed some issues that arose during the event. A well-known question came into our focus: "Why is it easier to pull a sledge horizontally than to pull it on a slope upwards?" (Fig.1.)

Fig.1. Pulling a sledge

Some of the students paid special attention to the problem and showed interest to participate in a deeper study. I chose the Aristotelian (often called peripatetic) way of education to mentor or guide this little group. It is a teaching method that roots in the ancient times and points towards Inquiry Based Learning. This method builds on the students' skills, also reinforces these. It is open between subjects. It supports inductive teaching, gives an active and constructivist approach of learning. It is one of the most motivating and effective ways of education, it also matches best the age characteristics of the high school students. Our group achieved surprising results in the study.

A STUDY OF THE TYPICAL QUALITATIVE ANSWERS

We met answers in the Hungarian literature of physics competitions and methodology that are similar to this reasoning: "In both cases we need to exert a force against friction, plus on the slope we must exert force against a component of gravity as well." The problem is as follows: We agree that the force against gravity is an increasing value as the tilt angle of the slope is increasing, whereas the force against friction is a decreasing one. When we add up these two, the sum is not necessarily an increasing function.

Some of the answers we read are similar to this: “Besides energy dissipated in friction, extra mechanical work must be done to give “height”/ “positional”/ “potential”/ “gravitational” energy.” When talking about an “easier pull”, we associate it with forces rather than with energy. Work, energy, and force are different notions. If we want to make a connection, we need to study distance as well.

THE NEWTONIAN ANALYSIS

We used Newton’s laws, which are also well known as basics of classical dynamics for the theoretical analysis of the case. Our solution is often studied also in upper secondary physics courses. We apply the standard notation of dynamics and use the symbols F, m, a, μ, α, etc. Quantities characterizing pull on a horizontal surface are marked by *. Vectors are set in bold.

Based on Newton’s 2nd law the force needed for a uniform motion…

… in case of horizontal pull is as follows:

$$*\boldsymbol{F}_{\text{pull}} = -*\boldsymbol{F}_{\text{friction}} \quad (1)$$

since

$$\sum * \boldsymbol{F} = 0\,, \quad (2)$$

so

$$*F_{\text{pull}} = \mu \cdot m \cdot g \quad (3)$$

… in case of pulling up on a slope (see Fig.2.) is as discussed below:

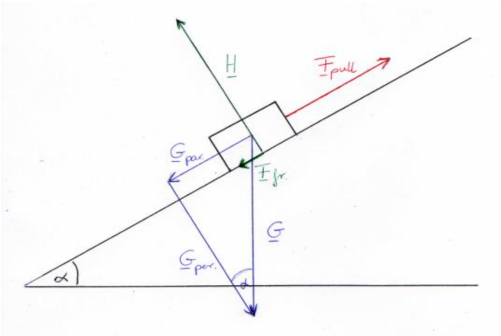

Fig.2. Study of forces on a slope

Newton’s 2nd law is a law of vectors. Often we need to use two simultaneous equations for the components. These are for the components parallel and perpendicular to the surface of the slope. The + directions are perpendicularly away from the surface of the slope, and crosswise up.

- Studying the components perpendicular to the surface provides **H**, the force that is exerted by the slope

$$\mathbf{H} = -\mathbf{G}_{\text{perpendicular}} \quad (4)$$

therefore

$$\mathrm{H} = \mathrm{m} \cdot \mathrm{g} \cdot \cos\alpha \quad (5)$$

- So we can calculate friction

$$\mathrm{F}_{\text{friction}} = \mu \cdot \mathrm{H} \quad (6)$$

so

$$\mathrm{F}_{\text{friction}} = \mu \cdot \mathrm{m} \cdot \mathrm{g} \cdot \cos\alpha \quad (7)$$

- The parallel component of gravity can be given as

$$\mathbf{L} = -\mathbf{G}_{parallel} \tag{8}$$

that means
$$L = m \cdot g \cdot \sin\alpha \tag{9}$$

Based on Newton's 2nd law the force of pull is

$$\mathbf{F}_{pull} - \mathbf{F}_{friction} - \mathbf{L} = \mathbf{0} \tag{10}$$

which gives us that
$$F_{pull} = \mu \cdot m \cdot g \cdot \cos\alpha + m \cdot g \cdot \sin\alpha \tag{11}$$

$$F_{pull} = m \cdot g \cdot (\mu \cdot \cos\alpha + \sin\alpha) \tag{12}$$

To compare the force of pull in these cases we form a function

$$\psi = F_{pull} - {}^{*}F_{pull} \tag{13}$$

We receive that
$$\psi = m \cdot g \cdot (\mu \cdot \cos\alpha + \sin\alpha - \mu)\,. \tag{14}$$

If we study the sgnψ function, we can figure out whether our original statement is true or false. We need to face a problem: analysing a function like sgnψ is not in the secondary school curriculum. A few decades ago we could not have coped with this analytical task.

NUMERICAL ANALYSIS, A STUDY OF THE SGN Ψ FUNCTION

Two of the students participating in the project were senior students of a secondary IT software course at the time. Relying on their choice we wrote a programme in C++ using SDL, which works in 1000x180 pixels [1]. Since $0^o \leq \alpha \leq 90^o$, on the vertical axis we can easily represent the tilt angle, α if 1^o=2 pixels. So, on the horizontal axis we can represent μ. With a multiplier we can adjust the maximum value to what we want to study. Our programme works in two cycles. This means 90,000 data pairs to calculate with. We presented the results according to our purpose in a colour code (see Table 1.)

Table 1. Colour code

Pull on slope	Pull on level ground (*)	sgnψ	Colour code
bigger	smaller	+	red
smaller	bigger	-	blue

We were very excited to see the results. If a blue area appears, it means that the original statement is not necessarily true in all circumstances. Our results in the numerical analysis:

1) For all possible angles, if $0 \leq \mu \leq 0.25$ see Fig.3.

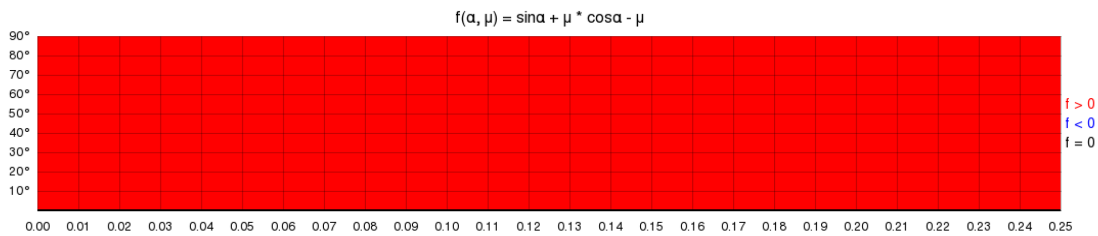

Fig.3. Sign for small μ values

2) For all possible angles, if $0 \leq \mu \leq 2.5$ we present Fig.4.

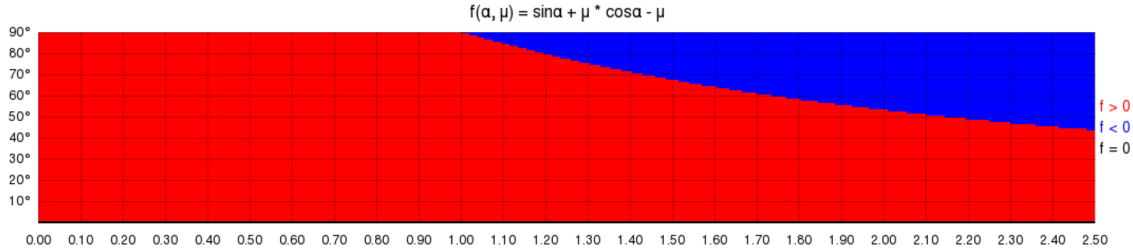

Fig.4. Sign for bigger μ values

3) For all possible angles, we allowed μ up to 50, you can check Fig.5.

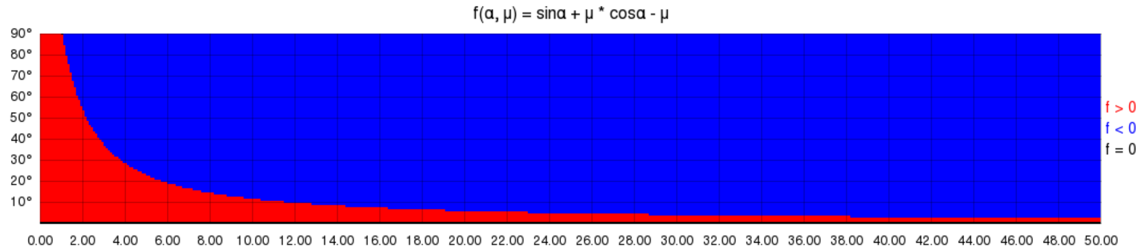

Fig.5. Sign up to extreme μ values

We can conclude that α and μ are the main quantities that define motion on a slope. The blue area appears only if $\mu > 1$. We left the question open whether there is a significance in physics of $\mu = 1$.

HANDS-ON MEASUREMENTS

We wanted to see what the typical values (for μ and α) are, when riding the sledge.

Measuring the coefficient of friction

We pulled the sledge on level ground at constant speed. We used an 80213-141 Kamasaki digital scale (dynamometer) that we bought cheap in a fishing shop. We also needed a bathroom scale and a sledge. We made measurements on 3 different occasions, which means 3 different circumstances. We decided to note 3 readings each time. We formed the mean value by calculating the arithmetic mean. Our results are in Table 2.

Table 2. Our results for the coefficient of friction

measurement (Budapest XXI. ÁMK)	$F_{gravity}$ (N)	F_{pull} (N)	$\mu = \frac{F_{pull}}{F_{gravity}}$	μ_{mean}
9th Febr. 2015 late evening with a young girl on	351+51.7= 403	45.15	0.112	0.118
		49.46	0.123	
		47.88	0.119	
10th Febr. 2015 afternoon	51.7	9.88	0.191	0.178
		9.20	0.178	
		9.45	0.166	
16th Febr. 2015 early morning	51.7	4.90	0.095	0.092
		5.10	0.098	
		4.35	0.084	

In the 1985/5. issue of the Hungarian journal “KÖMAL” we found that the friction constant measured with a different method is $0.02 \le \mu \le 0.3$. Our results match those we found in the literature.

Measuring tilt angles in 2 ways

Our first problem was that we could not get an inclinometer. Since this instrument is not cheap, we worked out a conventional method for measuring α. We needed a bubble level (0.8m), a 1-meter rod, and a pendulum (string & load). Figs.6. and 7.demonstrate how we used our tools. We also used another apparatus to measure tilt angles, the GPS system. We worked with the two versions that were available free.

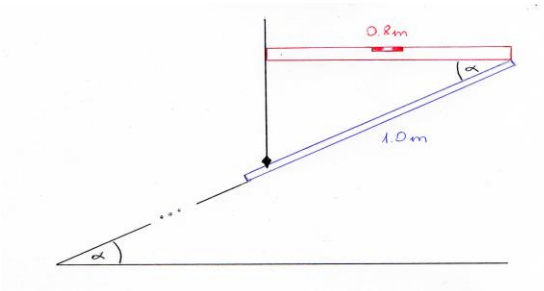

Fig.6. Our “inclinometer”

We made our measurements in different playgrounds in Budapest on 23rd June 2015.

Fig.7. “In-situ” measurements

Table 3 contains our results. We denote by * our results with the GPS system.

Table 3. Our results for α

	spot	$l_{projection}$ (cm)	$\cos\alpha$	α_{actual}	α_{mean}	$^*\alpha_{actual1}$	$^*\alpha_{actual2}$	$^*\alpha_{mean}$
Slope 1 Bp., 1095 Petőfi u. 2.	1/1	84.0	0.9524	18^o	**15^o**	16^o	13^o	**15^o**
	1/2	85.0	0.9512	20^o				
	1/3	80.5	0.9938	6^o				
Slope 2 Bp., 1091 Kékvirág u. 2.	2/1	80.5	0.9938	6^o	**11^o**	11^o	14^o	**12^o**
	2/2	81.5	0.9816	11^o				
	2/3	83.3	0.9639	15^o				
Slope 3 Bp., 1107 Bihari u. 3-5.	3/1	83.5	0.9581	17^o	**17^o**	15^o	14^o	**15^o**
	3/2	85.0	0.9412	20^o				
	3/3	82.5	0.9697	14^o				

Our results range from 6^o to 20^o, and a characteristic (mean) value is 14^o. We consider our result as a good estimate only, based on “in-situ” measurements.

INCORPORATING THE RESULTS OF OUR THEORETICAL AND PRACTICAL STUDIES

„Why is it easier to pull a sledge on level ground than to pull it up a slope?"

We provide two answers:

1) Since $\mu < 1$, from the theoretical study we learned that there is no need to give a typical value to α. (Fig.8.) A correct answer is: As typically $\mu < 1$, it is easier to pull a sledge on level ground than to pull it up a slope.

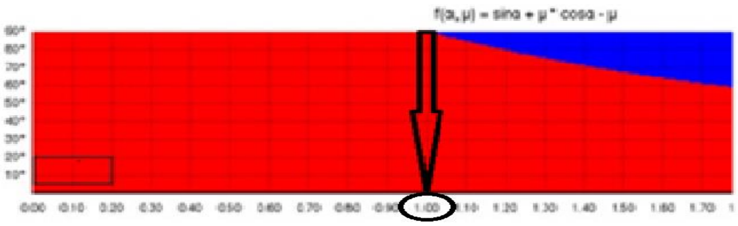

Fig.8. The case when $\mu < 1$

2) We studied the area denoted by the typical values based on our measurement (Fig.5.). Another correct answer is: It is easier to pull a sledge on level ground than to pull it up a slope, because of the real values of α and μ.

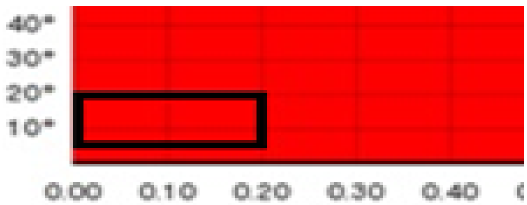

Fig.9. Real sledging values

CONCLUSIONS

In a mentor class the teacher and the students studied a question that had been studied before only in a qualitative way in public education. First, we have found that the typical answers in this case are not correct. Then we asked a "why?" question to a "yes-or- no" type question. Easily we acquired a function of two variables. From this point on computing skills were needed to carry on with our project. Our mentor class turned into an interdisciplinary forum of science. We could find real answers for a classical physics question, where the qualitative answers based on experience do not capture the scientific essence of the phenomenon.

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