CAROUSELS TO CORIOLIS, OR HOW PHYSICS SUPPORTS UNDERSTANDING GEOGRAPHY

Andrea Gróf
Karinthy Frigyes Gimnázium, Budapest, Hungary, gandi@karinthy.hu
Physics Education PhD Program, Eötvös University, Budapest

ABSTRACT

There is a conflict between the ways motions are described in physics and geography classes. While non-inertial frames do not feature in official physics curricula, geography texts rely on inertial forces in explaining motions of the atmosphere and the seas. Prompted by a survey demonstrating that the physical principles behind geography are not understood, this paper presents a possible treatment within the limits of high-school mathematics. Through the classic example of a merry-go-round, inertial forces are introduced quantitatively, and the results are applied in problems related to motions in geography.

INTRODUCTION

The choice of reference frame is a central idea in the physics class, while in geography they just use their “natural” frame without addressing the issue of reference frame at all. Furthermore, that frame is a non-inertial one, whereas we at most switch from one inertial frame to another, and may even reproach our students if they dare to say “centrifugal force”. When geography is taught in year 9, underlying physical concepts and principles are either lacking, or recently acquired knowledge is not yet supported by sufficient experience. Explanations given by geography texts are often superficial or even wrong, but the conflict exists even in the case of a correct approach. With more background knowledge and expertise in problem solving, it is worth revisiting geographic phenomena in physics lessons later on.

A SURVEY ON PHYSICS BEHIND GEOGRAPHY

A multiple choice survey with 215 students revealed a serious lack of understanding, with no significant difference regarding whether they had completed geography before physics, or they had studied both subjects in the same year. The survey encompassed a wide range of concepts related to timekeeping, the shape of the Earth, motions of air and the seas, etc. Two of the questions involving inertial forces are shown below.

One question tested the understanding of the nature of such forces: “The oblate shape of the rotating Earth is generally explained in terms of the centrifugal force. On the other hand, in physics problems dealing with circular motion and rotation, no centrifugal forces were considered. What is the difference?” The correct answer of different frames was only chosen by 17%. Distractor answers (based on classroom experience, and possible misinterpretations of geography texts) would deserve deeper analysis but that is beyond the scope of this paper.

Another question addressed the perennial myth of the kitchen sink: “Ivan in Moscow and Pedro in Buenos Aires each fill the kitchen sink with water and remove the stopper. The water drains with a whirl. What will they observe?” Only 10% gave the correct answer. The most popular distractor (56%) was the one stating that water whirls counter-clockwise for Ivan and clockwise for Pedro. This suggests that students learn their geography regarding cyclones, and apply it without criticism to anything that rotates. Just like Sylvester Stallone in Escape Plan, observing the toilet and concluding that the prison is on the southern hemisphere.

The myth is reinforced by "demonstrations" of the Coriolis force performed to tourists at the Equator, showing how draining water whirls one way or the other if the apparatus is moved a few metres to the north or to the south. They all cheat since the horizontal Coriolis force is zero at the Equator and varies as the sine of the angle of latitude. See [1] for an amateur video to observe angular momentum created by pouring water in a sink from the appropriate side. Tourists give credit to the presentation, although deception is quite apparent. It is instructive to reproduce the "demonstration" in class. (Just draw a random line on the floor and call it the Equator.)

INTRODUCING INERTIAL FORCES

The merry-go-round is a standard illustration of a rotating reference frame. However, high-school level resources normally offer conceptual treatment only. The approach demonstrated here is quantitative, without resorting to any vector calculus or even a vectorial product. Since it applies a lot of the dynamics taught in the regular curriculum, it can be used as a kind of synthesis, adding a little extra at the end of the year.

The programme features a rotating observer A whose reference frame is attached to the centre and rotates along with the roundabout, an inertial observer B, and two further characters: a lizard running along the rim, and a sparrow scared away and flying radially (Fig. 1). Numerical values are calculated in each case, to give an idea of how various forces or accelerations compare to each other.

Suppose the mass of A is 20 kg, the radius of the carousel is 1.5 m, and it completes a revolution in 3 s. As seen by B, the speed of A is then v = 3.14 m/s, and she is acted on by a net force of $m\omega^2 r$ = 132 N. It is important for students to understand that this is the inward push by the merry-go-round seat, and since it is a real force exerted by a real object, the same force must be present in A's frame, too. Since A is in equilibrium in A's frame, that raises the need for an extra outward force of magnitude $m\omega^2 r$, so the centrifugal force is introduced.

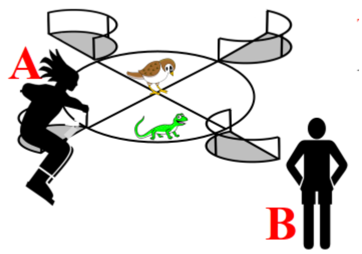

Fig.1. The four characters

Next, the motion of the lizard is considered in each frame. Assume its mass is 20 g and it runs at a speed of u = 0.50 m/s along the rim. Again, the observers must agree on the force exerted by the merry-go-round. That constitutes the net force for observer B (Fig. 2, top left):

$$F_{\text{net}} = F_{\text{merry}} = ma = m\frac{(v+u)^2}{r} = 0.0200 \cdot \frac{3.64^2}{1.50} = 0.177\,\text{N}\,. \qquad (1)$$

For rotating observer A, however, the speed of circular motion is only 0.50 m/s, and the net force is only 0.003N, so outward forces need to be added to the merry-go-round force of 0.177 N to produce a resultant of 0.003N. One such force is the centrifugal force of $m\omega^2 r$ that is calculated to be 0.132 N, but that alone will not produce the required resultant. Yet another outward force of $-0.003+0.177-0.132=0.042\,\text{N}$ is needed. What is the physical law behind that? Let us examine the forces and accelerations algebraically. Expand the square in (1):

$$F_{\text{merry}} = m\frac{(v+u)^2}{r} = \frac{mv^2}{r} + \frac{2mvu}{r} + \frac{mu^2}{r}\,. \qquad (2)$$

The last term of (2) represents the net force for A, a resultant of real and inertial forces:

$$\frac{mu^2}{r} = F_{\text{merry}} - \frac{mv^2}{r} - \frac{2mvu}{r}\,. \qquad (3)$$

The first term of (3) is the inward real force of the seat, the second term is the centrifugal force outwards, and the last term is the missing force, also outwards this time. Thus an object moving tangentially at speed u is acted on by a force $2mvu/r = mu \cdot 2\omega$. This is the Coriolis force, and substitution of numerical data yields the magnitude of 0.42 N.

Figure 2 below summarizes the forces in the two frames. The direction of the Coriolis force is the opposite when the lizard is running the other way. A special case of this situation occurs when A observes the motion of B, who stands on the ground, a distance R from the centre. According to A, he is moving in a circle at a tangential speed of $u = -\omega R$, that is, his (net) acceleration is $a = \omega^2 R$. Since there is no real horizontal force exerted by other objects on him, this acceleration is caused by the two kinds of inertial forces: the outward centrifugal acceleration $\omega^2 R$, and an inward Coriolis acceleration of $2\omega u = 2\omega^2 R$. So the resultant is $a = 2\omega^2 R - \omega^2 R = \omega^2 R$ inwards.

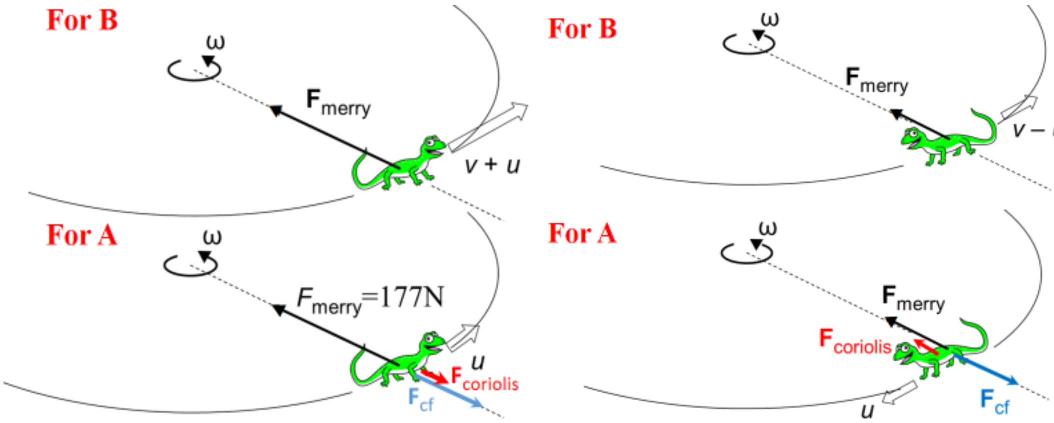

Fig.2. Forces on an object moving tangentially.
Left panel: in the direction of rotation, Right panel: opposite to the direction of rotation

So far, we have investigated objects in tangential motion and radial forces acting on them. The motion of the sparrow flying away from the centre is uniform and radial for B, but rather complex from the point of view of A. Radial acceleration is zero in A's frame, too, (like for the motion of B in the previous example,) but the tangential speed is increasing in proportion to the distance, so this time there is a tangential force, too. Figure 3 (right panel) shows the constant radial speed and increasing tangential speed of the sparrow at equal intervals of Δt.

If the distance from the centre increases by Δr in a time Δt, then $v = \Delta r/\Delta t$. In a short time Δt, acceleration can be considered uniform, angular displacement increases by $\omega\Delta t$, which means a distance of $\omega\Delta t \cdot \Delta r$ covered in a direction perpendicular to the radius. That is,

$$\frac{1}{2}a(\Delta t)^2 = \omega\Delta t \cdot \Delta r\,.$$

$$a = \frac{\Delta r}{\Delta t} \cdot 2\omega = v \cdot 2\omega\,.$$

The same formula is found to apply to the sideways force due to radial motion as to the sideways force due to tangential motion. Hence, it applies to every motion in a plane perpendicular to the rotation axis. The treatment of the Coriolis force is completed.

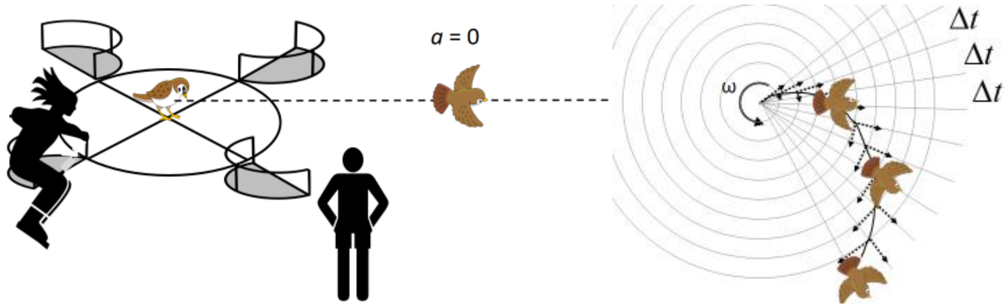

Fig.3. The motion of the sparrow as it appears to B (left panel) and A (right panel)

APPLICATIONS ON THE ROTATING EARTH

The selection below gives some outlined and some worked examples of quantitative exercises related to geography. Note that the angular speed of the Earth is $\Omega = 7.292\cdot10^{-5}$/s.

Exercise 1. Free fall acceleration is the resultant of gravitational acceleration towards the centre and centrifugal acceleration away from the axis. Thus, g is found to be 9.78 $\mathrm{ms^{-2}}$ at the Equator. The value of g influences sports results: for example, if an athlete can jump 8.00 metres at the poles then, assuming the same initial speed and angle, his jump is calculated to be 4.04 m at at the Equator.

Exercise 2. Budapest lies at a latitude of N47.5°. Find the magnitude and direction of the centrifugal acceleration and of the free fall acceleration at Budapest. Calculate with the average radius of the Earth, R = 6370 km.

Solution. $a_{cf} = \Omega^2 \cdot R\cos\varphi = (7.29\cdot10^{-5})^2 \cdot 6.37\cdot10^6 \cdot \cos 47.5° = 0.023\frac{\mathrm{m}}{\mathrm{s}^2}$,

directed away from the axis of rotation. The magnitude of the vector sum (Fig. 4) with the gravitational acceleration towards the centre is obtained by using rectangular components:

$$a_g = \gamma\frac{M}{R^2} = 6.672\cdot10^{-11}\cdot\frac{5.974\cdot10^{24}}{(6.370\cdot10^6)^2} = 9.823\frac{\mathrm{m}}{\mathrm{s}^2}.$$

$$g = \sqrt{(9.823\cdot\cos 47.5° - 0.023)^2 + (9.823\cdot\sin 47.5°)^2} = 9.81\frac{\mathrm{m}}{\mathrm{s}^2}.$$

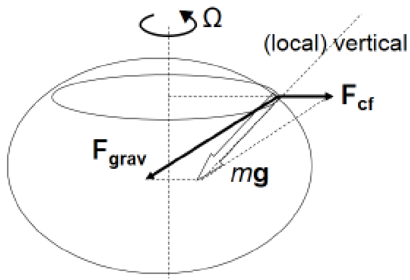

Fig.4. The direction of free fall acceleration

Its direction is somewhat to the south of towards the centre. This is what “down” means; the flattened shape of the Earth formed to make the surface perpendicular to this direction.

Exercise 3. To link with the merry-go-round example, the Coriolis force should first be investigated at the Equator. Suppose wind is blowing at a speed of u = 20 m/s towards the west at the Equator. The Coriolis acceleration is found to be $2\Omega u = 2.9\cdot10^{-3}$ $\mathrm{ms^{-2}}$, directed vertically downwards. Note that its direction is radial, that is why the whirling water “demonstrations” are hoaxes.

Exercise 4. At other latitudes the Coriolis force has a horizontal component, too. Since we consider motions in a plane perpendicular to the local vertical rather than to the axis, the situation is more complex than the carousel case. High-school texts normally refer qualitatively to Foucault's pendulum as demonstration, but they do not explain the value of the local angular speed. By a high-school level adaptation of the explanation (based on the transport of vectors on curved surfaces) offered by some advanced texts (e.g. [2]), the use of angular velocity vector and components may be avoided: Students know that the surface of a cone unfolds in a plane. Consider the cone touching the globe along the φ = N48.8° parallel of Paris (Figure 5). In one day, while the Earth turns through 2π, Paris (point P) only turns in the unfolded plane by an angle of $2\pi\cdot\sin\varphi$. Hence the local angular speed is $\sin\varphi$ times that of the Earth:

$$\omega = \Omega\cdot\sin\varphi = 7.29\cdot10^{-5}\cdot\sin48.8° = 5.49\cdot10^{-5}\,/\,\mathrm{s}$$

which means 11.3° per hour. It would be 15° at the poles and zero at the Equator.

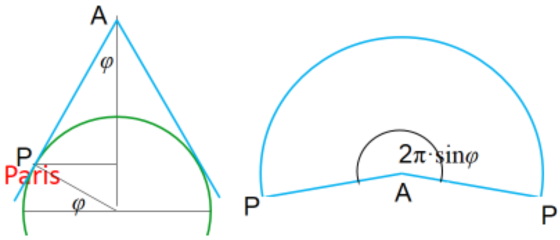

Fig.5. Demonstration of turning in a plane perpendicular to the local vertical

Exercise 5. (a) The mystery of the kitchen sink unravelled at last. Calculate the acceleration of a bread crumb in Budapest, circling at a radius of 2 cm, at a speed of 10 cm/s. What part of the acceleration is due to the Coriolis force? (b) Answer the same question for Jupiter's great red spot, a giant whirlwind at S22° latitude, angular size 25° by 12°. Wind speed is in the order of 100 m/s. The radius of Jupiter is about 72 000 km, and it rotates fast, completing a revolution in 9.8 hours. (Based on [3].)

Solution. (a)
$$a = \frac{v^2}{r} = \frac{0.1^2}{0.02} = 0.5\,\mathrm{m/s^2}.$$

$$a_C = 2v\Omega\sin\varphi = 2\cdot0.1\cdot(7.3\cdot10^{-5})\sin47.5° = 1\cdot10^{-5}\mathrm{m/s^2}.$$

The effect is very small compared to other effects responsible for the motion, such as the geometry of the sink or the initial angular momentum that the water happens to have.

(b) 1° on Jupiter corresponds to $2R\pi/360 = 1.2\cdot10^6$ m, so the roughly 9° radius of the spot means an acceleration of about $9\cdot10^{-4}$ ms^{-2}, while the Coriolis acceleration is found to be about $1\cdot10^{-4}$ ms^{-2}. It is comparable to the net acceleration, so the Coriolis force does play a role in the formation of this persisting storm.

Exercise 6. (a) A golfer in Scotland, N55° latitude, can hit the ball to 300 m at a 45° angle. (b) An artillery missile is launched at 700 m/s. Does the deviation owing to the Coriolis force need to be considered? (Calculate the sideways deflection owing to the Coriolis force.)

Solution. (a) Using the known formulae of projectile flight, a range of 300 m implies an initial speed of 54 m/s, and a flight time of 8.7 s. Hence

$$d = a_C\frac{t^2}{2} = (2\cdot\Omega\sin\varphi\cdot v_0\cos\alpha)\frac{t^2}{2} = 7.3\cdot10^{-5}\cdot\sin55°\cdot54\cos45°\cdot8.7^2 = 17\,\mathrm{cm}.$$

The deflection is probably negligible compared to other disturbing effects like wind.

(b) The range is now 50 km, and the sideways deflection is about 300 m. This time, the effect is significant, it has to be considered in aiming the missile.

Exercise 7. What happens if a hockey puck is hit in a perfectly frictionless ice rink? Not straight line motion! The net force on the puck is not zero in the rotating reference frame of the Earth. If the ice is perfectly horizontal, that is, perpendicular to the local vertical, all other forces will cancel, leaving the horizontal Coriolis force, a sideways force as resultant. That leads to circular motion.

Exercise 8. How fast should we hit the puck in Budapest so that the circle fits in an ice rink 30 m wide? (Based on [3].)

Solution.

$$a_{\text{centripetal}} = \frac{v^2}{r} = 2\Omega \sin\varphi \cdot v\ .$$

$$v = 2\Omega \sin\varphi \cdot r = 2 \cdot 7.3 \cdot 10^{-5} \cdot \sin 47.5° \cdot 15 = 0.61 \frac{\text{mm}}{\text{s}}\ .$$

Quite slow. For speeds in the order of a metre per second, we need r = 9.3 km in Budapest (39 km at 10°, and 7.0 km at 80° latitude). Such large ice rinks we do not have, but nature realizes this kind of motion. Figure7 below shows the positions of a buoy in the Baltic sea, southeast of Stockholm at N57° latitude [4],[5].

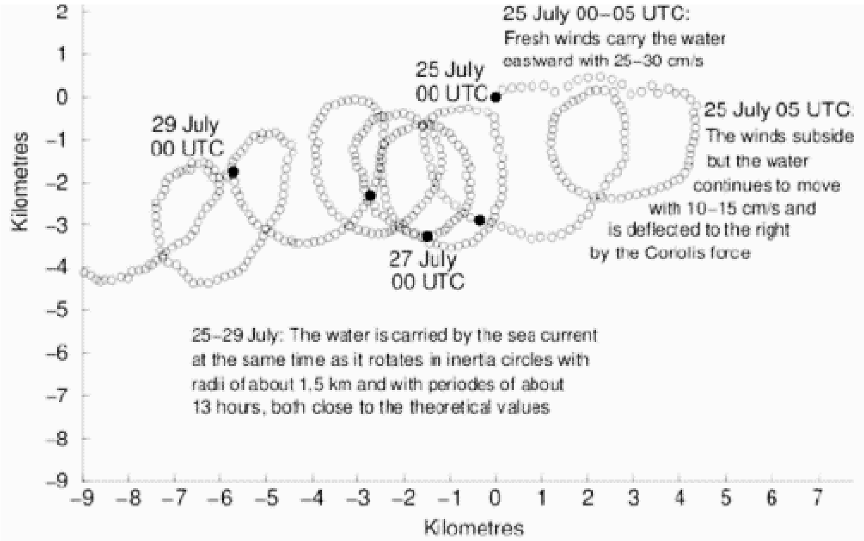

Fig.7. A buoy at sea executing inertial motion [3], [4]

CONCLUSION

Quantitative treatment (algebraically as well as with numerical magnitudes) helped decide whether inertial forces should be considered or can be ignored in a particular situation. Application to geography-related problems supports a deeper understanding in both subjects. As indicated by the results of a short quiz, the investigation of the same motion from different points of view made students more conscious of different reference frames.

REFERENCES

1. https://www.youtube.com/watch?v=4IIVfoDuVIw
2. V. I. Arnold: *Mathematical Methods of Classical Mechanics*, Springer-Verlag, New York, NY, 1978
3. A. Münchow: *Geophysical Fluid Dynamics*, University of Delaware, Newark, 2005
4. A. O. Persson: The Coriolis Effect: Four centuries of conflict between common sense and mathematics, History of Meteorology **2**, 1, 2005
5. http://www.cleonis.nl/physics/phys256/inertial_oscillations.php