GROUP DECISION TAKING, AS VIEWED THROUGH SOCIOPHYSICS

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ABSTRACT

After a short introduction to statistical physics, the Ising model, and its Monte Carlo simulation, we discuss the insight offered by such modelling to decision taking in small social groups.

INTRODUCTION

Sociophysics is a relatively new branch of statistical physics that became a field of active research in the last decades. The name has been given to it by the French physicist Serge Galam, eventually called the "father of sociophysics" [1]. Below we introduce sociophysics through the example of how a community takes some decision; we would like to convince our readers and through them their students, too that a well-chosen physical analogy, in this case the Ising model of magnetic ordering, may offer some unexpectedly interesting insight to such a human game.

We begin with a quick look at the broader physical background. Statistical physics came into being in the second half of the 19th century, as serious indications appeared that matter around us is composed of tiny particles – atoms, molecules – interacting with each other and their environment. Because of their small sizes, all we see as material properties is actually expressing a statistical average of an enormous number of atoms, molecules, or their still smaller components, like electrons in a metal.

By now all that became one of the most extensive fields of physics, describing a wide variety of phenomena, using a huge mathematical apparatus [2]. However, it has a simple core that can be thought in the secondary school, requiring only some acquaintance with the exponential function. Maxwell was the first to calculate the velocity distribution of atoms in an ideal gas, flying around, eventually colliding and exchanging energy and momentum among themselves. That was extended by Boltzmann to the case when the atoms, still statistically independent, are moving under the action of an external force, controlling only the sum of their kinetic and potential energies; he was the first to write down that the probability for an atom to be in a given state (place and speed) depends only on its energy *E* in that state, and the absolute temperature *T*, in the form

$$f(E) = \frac{e^{-\frac{E}{kT}}}{Z} \tag{1}$$

("Boltzmann distribution"), where $k = 1.38 \cdot 10^{-23}$ J/K is the Boltzmann constant, whereas *Z* is a normalization constant ("partition function"), assuring that the sum of the probabilities of all possible states should be 1 since the system certainly has some state. Looking at the above formula, one sees that matter prefers to be in the states of smaller energy, however, high temperature is against such a statistical selection by energy; everything is determined by the ratio *E/kT*.

A major step forward was taken by Gibbs who discovered that the above formula refers not only to ideal gases but also to strongly interacting systems; however, then *E* denotes the energy of the whole system, not just an atom of it. Of course, for statistics to make sense, we have to imagine the system in many copies ("Gibbs ensemble"), or since to actually manufacture or just buy many copies would be too expensive, let it be just one of the copies, but do the same measurement a number of times on it, and evaluate the statistics of the many measurements.

One of the simplest examples for it, the Ising model explaining ferromagnetism, offers good chance to present the satistical physics way of thinking at high school and give it a quick test. Let us take a permanent magnet, e.g. a cube of magnetized iron, and imagine we are able to look inside and see its atoms carrying magnetic spins. According to the model, the magnetization of the iron cube depends on whether these spins are arranged to point in the same direction; those materials are called ferromagnets (iron, cobalt, nickel, and a few insulating crystals), for which that ordering happens spontaneously. It has been discovered by Pierre Curie that on heating up, the spontaneous magnetization decreases, and at a sharply defined temperature ("Curie point") it vanishes.

The source of spontaneous magnetization is the interaction between atomic spins; the simplest model for it, now called Ising model, has been presented in the PhD thesis of Ernst Ising. According to the model, a spin can take just two directions: up or down; for the *i*-th spin that is described by the number S_i = +1 or -1. The energy *E* of the system of interacting spins depends on the orientations of all spins, as described by the formula

$$E = -\sum_{(i,j)} J_{ij} S_i S_j - H \sum_i S_i \,, \qquad (2)$$

where J_{ij} is the coupling between the *i*-th and *j*-th spin, *H* being the external magnetic field. If each J_{ij} is positive (that makes a material ferromagnetic), without external field (*H*=0) the energy will reach its minimum if all the spins are oriented in the same direction – it doesn't matter which, $S_i S_j$ is positive anyway. That kind of ordering is found by the material, if the temperature is not too high.

As for people, like atoms or their spins, there are a lot of them, and they can be coupled to each other and their environments in various ways. You can call it an analogy between the two kinds of systems, although interactions between people are of course qualitatively different from those between atoms. That cannot stop us from applying the methods of statistical physics to social systems. Of course, human relationships are by far more complex. Social processes are determined by individual as well as collective behaviour. To study the latter, one needs insight into human relationships, a major subject of sociology. Similarly to statistical physics, where one is not interested in individual atoms, just their collective behaviour, sociologists study the collective behaviour of people. In the case of small groups – students in a class, or inhabitants in a condominium – collective behaviour is often dominated by pairwise interactions between people, which gives us a chance to use Ising-type models to get insight into the life of human communities.

Forces exerted by one atom on another depend on the distance between the atoms, as well as some of their physical properties, like electric charge, magnetic moment, or mass. Irregular motion of atoms is characterized by temperature: warming up means faster motion, which disturbs the ordering effect of forces. Analogously, "faster motion" of people means they are subject to many side influences, "they have little time" to pay attention to each other; in collective behaviour that means weaker interaction (weaker communication, less attention); they follow many other things, loosing contact to each other.

The environment influences a system not only through the noise of thermal motion; sometimes it exerts ordering effect on the system. For atoms, that can be an external electromagnetic field or gravity; in ferromagnetic ordering, an external magnetic field. For human communities, that can be a dominant personality, or some special knowledge of a member of the group.

Returning to the example of magnetic ordering, with Equation (2) in view, one can use Equation (1) to calculate the probability of a given configuration of spin orientations; all you need for it is the absolute temperature *T*. Once you know the probabilities, you can calculate mean values (statistical averages), e.g. the mean value of the magnetization $M = \sum_i S_i$ under vanishing magnetic field (*H=0*), and you can check under which critical temperature will the average magnetization be different from zero: that is, the temperature under which spins are oriented in one of the possible directions with high probability, which makes a magnet be a magnet.

If spins are arranged into a one-dimensional chain and only neighbours are coupled by the same positive coefficient *J*, it is easy to calculate everything in simple formulas, as done in Ising's thesis; however, the results are not too exciting. For the case of spins sitting on a two-dimensional lattice, Onsager carried out the same calculation in a horribly complicated way; don't worry if you have never learned how. In three dimensions, there is no solution in the form of closed formulas, however, there are efficient methods of numerical calculations; that is what we propose below for our readers and their students.

COMPUTER SIMULATIONS

Since we want to follow the probabilities of random motions, we use the Monte Carlo method, applying random numbers [3,4]. That general method can be applied by students to obtain interesting results by not too complicated tools. Within the Monte Carlo method, we apply the ingenious Metropolis algorithm that starting from some initial spin values, iteratively visits all spins in a lattice, repeating several times; individually flips a spin or not (see below); after each round calculates the magnetization, and repeats all that until reaching thermal equilibrium, in which – apart from tiny fluctuations – the magnetization does not change any more.

Whether in a given round a given spin flips over or not is decided within the Metropolis algorithm in the following way:

- assuming that the actual state of the spin has flipped from -1 to +1 or from +1 to -1 resp. (according to which state it has been found in), from Equation (2) we calculate the energy change ΔE accompanying the flip, which determines the acceptance probability *W*:

$$W = \begin{cases} 1 & \text{if} \quad \Delta E < 0, \\ e^{-\frac{\Delta E}{kT}} & \text{if} \quad \Delta E \geq 0, \end{cases} \tag{3}$$

- generate a random number between 0 and 1, and decide if we accept the assumed flip or not:

accept if the random number $\leq$ *W*,
reject if the random number $>$ *W*.

The easiest way to illustrate the Metropolis algorithm is through Excel macros written in Visual Basic language. Students can write the program themselves; the results are seen immediately.

Let us take a square lattice of size 25×25, having one spin in each cell. Except those on the boundary, each cell has 4 neighbours, interacting with each of them by the same coupling constant *J*, and not coupled to non-neighbouring cells. The different behaviour of the boundary cells is responsible for surface energy; it is a complication for the calculation that can be avoided by applying a periodic boundary condition, turning the square lattice into a torus. It is easy to build that into the simulation program: the lower and upper boundary lines, as well as the left and right boundary columns, become neighbours to each other; thus each cell has 4 neighbours: the iteration can be carried out under the same conditions for each cell.

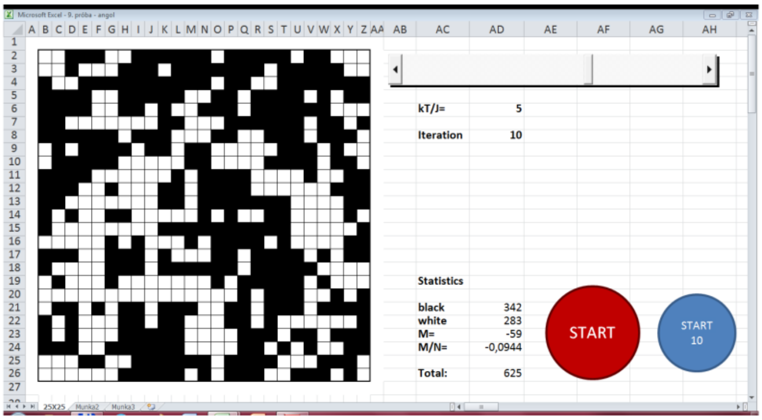

Fig.1. Excel worksheet for the simulation

It is easy to display spin values in each cell by painting the corresponding Excel cells white ($S=+1$) and black ($S=-1$). This simulation is not too fast (painting makes it still slower) but offers students the chance to follow visually as spins keep flipping during the steps of iteration. As output, we evaluate M/N, where M is the total magnetic moment of the square lattice (on the surface of the torus), whereas N is the number of cells: *625*. The value of the parameter kT/J can be changed by a scrollbar; k is Boltzmann's constant. As the initial state, we assign a random spin to each cell; spins are updated in each round of iteration according to the Metropolis algorithm (see Figs.1.,2.).

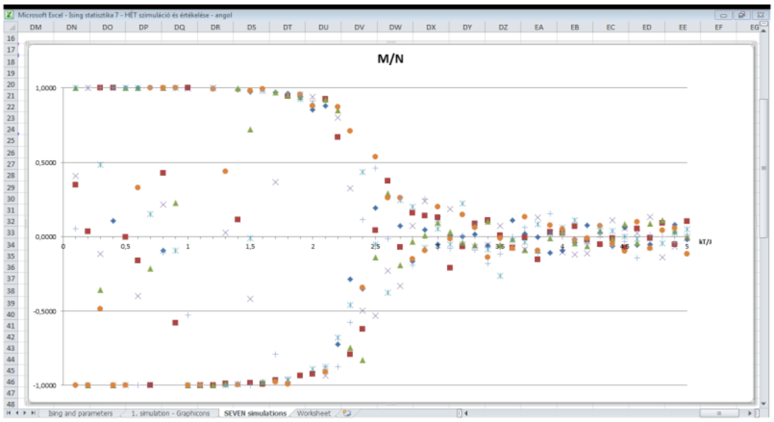

Fig.2. Spontaneous magnetization as a function of temperature, calculated from the Ising model by Monte Carlo method with no external magnetic field ($H=0$) in 7 different runs of 100-step iteration, kT/J changing from *0.1* to *5.0* in steps *0.1*. The phase transit-ion from ferromagnetic to paramagnetic state is clearly visible; its temperature agrees well with Onsager's prediction for the Curie point, $kT_c/J=2.27$. Similar calculations can be done with external magnetic field, causing non-zero magnetization for $T > T_c$, too.

THE SOCIOPHYSICAL APPLICATION: COLLECTIVE DECISION TAKING

Now we turn to our main topic: utilizing the model for the study of social systems. We recall that the Ising model can be applied in cases where there are but two options: "yes" and "no", corresponding to "spin up" and "spin down" [5,6]. A convincing collective "yes" or "no" decision would correspond to magnetic ordering.

A simple situation open to Ising-like modeling is this: the community of a school class has to decide whether their next excursion would be directed to a mountain or to a lakeside. They are assumed to decide by majority voting. Any decision is acceptable; no decision would be a trouble.

The basic analogy is clear: if the students pay attention to each other's preferences, it helps them in reaching a decision favourable to many of them. Still, in connecting the case to a ferromagnetic model, to choose the right parameters is the critical step. Majority voting corresponds to measuring magnetization: if +1 point is given for a vote for mountains, -1 for lakeside, the majority outcome is decided by the sign of the sum M. However, for a class of N students, it is the ratio M/N which characterizes how convincing the decision is.

In the magnetic model that depends on the temperature; more exactly, on the ratio kT/J. It is not easy to tell what "sociological parameter" would correspond to that ratio in our decision taking problem. The answer is not unique, so it is worth asking students what they think about it, how should one characterize the strength of the connections among them (e.g. the average number of a student's friends), and the factors detracting their attention from the question to decide (e.g. the number of topics they used to discuss among themselves in a break, or a forthcoming test exam). You may try various suggestions and compare the results. In the model there is one more parameter: the external magnetic field H. In the social case that might correspond to an extra possibility of sporting activity, or a cheap lodging at one of the possible destinations; eventually just the lack of it, which may influence the decision for some or many of the voting students.

Leaving the school, other minor social groups may get into similar situations. An example is a condominium – a house of separate flats, each owned by its inhabitants – where from time to time collective decisions have to be taken. If it is a binary decision, say, there is a hole on the roof: fix it now or save the money and put there a washbasin for some time, you can use an Ising-like model to analyze the situation. Again, the question is how to characterize the strength and character of the human connections, as well as the attention devoted to the particular problem, all influencing the decision. That is a problem of sociology, and physical modeling can help get insight to the outcome of the decision process. Again, there are "external fields": various factors influencing the opinion of each inhabitant.

While encouraging our readers to invent more and more examples of social decision taking, we emphasize the limitations; in particular, larger communities – like a bigger city – can take collective decisions on questions of extreme importance, like communal traffic; however, in questions of minor importance, it would fall into smaller units (districts or even streets) taking their own decisions on the same issue. If you just think how different we people are, it is very surprising to obtain any insight into social problems at all through simple models. A deeper reason for that can be that there is little cross-talk between the levels of organization: chemistry is played by electron clouds, utterly insensitive to fine properties of atomic nuclei; minor chemical differences between people play no role except under the extreme circumstances of transplantation, and in many, many questions of the society, the multiplicity of opinions matters only in the exceptional moments one counts which answer is given by more of us about some deliberately posed question. On occasion, a clever simplification successfully expresses the essential points of what happens, and results in functional models; sometimes – like in the case of failed economical predictions causing small or big collapses through the things omitted from the model – in sour disappointment: the world now and then reminds us that we are adult persons, so we should better not mix up fairy tales with reality. Still, simple models help us surprisingly often to get some orientation about various problems of our narrow or broader human enviroment, and good statistical physics analysis may eventually offer you some help that would be hard to get otherwise.

THE SYMMETRY ISSUE

In the Ising model the interaction is symmetric: spin "i" spin acts on spin "j" by the same strength J as "j" acts on "i". Human connections are usually not symmetric. Beyond physics, we have tried the model for asymmetric interaction (its theory is quite complicated, see [7]). We assume coupling *J* to the right and upwards, and *qJ* ($0<q<1$) to the left and downwards. Figure 3. is an illustration; the main property is that magnetization first seems to reach thermal equilibrium, then flips over in a random way. In human decision taking: once a decision seems to be taken, it may turn into the opposite if you keep on discussing.

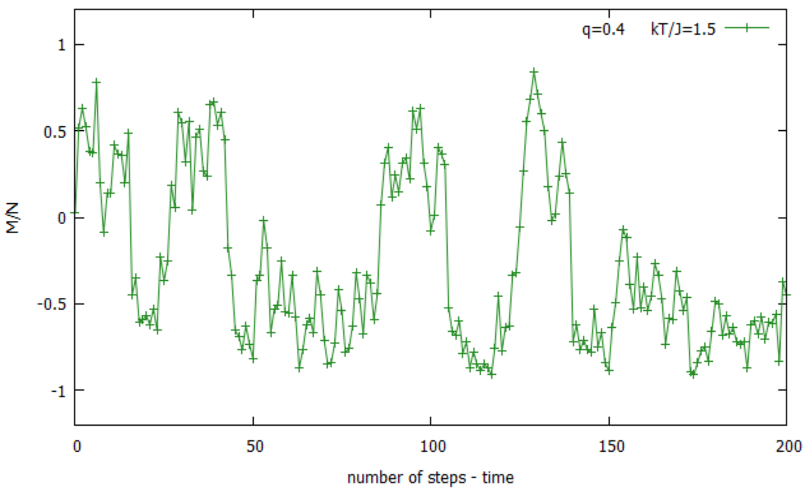

Fig.3. Time dependence of magnetization under asymmetric spin-spin coupling

CONCLUSIONS

Passages between remote fields of science are sometimes surprisingly fruitful, e.g. a lot can be learned about complicated social problems if you use simple physical models. That is the object of the recently opened branch of science called sociophysics. A relatively well clarified part of it is decision taking in small communities, to which one of the emblematic simple models of statistical physics, Ising's model of magnetic ordering, offers interesting insight. The topic can be taught in high school using simple numerical tools. Social phenomena are usually more complicated than the systems studied in physics, therefore attention should be paid to the limitations of a given model; an example is asymmetric coupling.

ACKNOWLEDGMENTS

This study has been inspired by the lectures of Zoltán Néda given in the "Physics Education" PhD program of Eötvös University. We also thank J. Cserti, J. Kertész and G. Szabó for their helpful suggestions to the first version of the present paper.

REFERENCES

1. S. Galam: Sociophysics, Int. J. Mod. Phys. C **19**, 409 (2008); http://arxiv.org/pdf/0803.1800v1.pdf
2. See e.g. Charles Kittel: Thermal Physics (available in several editions).
3. J. Cserti: Harnessed Hazard, Math. Phys. J. for Secondary Schools **3**, 48 (2004).
4. J. Kertész, J. Cserti, J. Szép: Monte Carlo simulation programs for microcomputers, European Journal of Physics **6**, 232 (1985).
5. D. Stauffer: Social applications of two-dimensional Ising models, Am. J. Phys. **76**, 470 (2008); http://arxiv.org/pdf/0706.3983v1.pdf
6. D. Stauffer: Statistical physics for humanities: a tutorial – http://arxiv.org/pdf/1109.2475v1.pdf
7. Crisanti A. and Sompolinsky H: Dynamics of spin systems with randomly asymmetric bonds: Ising spins and Glauber dynamics, Phys. Rev. A **37,** 4865 (1988).