PENDULUM WAVE OR LOVE AT FIRST SIGHT

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ABSTRACT

Pendulum wave is the motion of a chain of pendulums in which – if their length is chosen based on an appropriate mathematical relationship – remarkable shapes emerge. What should be the logic of composition? How long should the cords be in order for the pendulums to show nice shapes when they are initiated appropriately? We started to study this topic in more detail with high school students in a physics camp. Students had to develop their chosen project work in a team with the support of a teacher. After clarifying the physical and mathematical background, younger students prepared the tools, while the more senior ones worked on the related computer simulation that facilitated a more precise research.

INTRODUCTION

Few years ago we came across the beautiful phenomenon of pendulum wave, which was absolutely love at first sight. The students of our school dealt with this phenomenon in more detail for the first time in the 2014 physics school camp.

Fig.1. Snapshot of a pendulum wave (front view)

Let's see what the pendulum wave is. Pendulum wave is a series of pendulums in an optional number. If the length of each pendulum and the initial conditions are chosen by an appropriate mathematical relation, the pendulums can shape special formations. Our most important question is what the logic of composition should be. In other words, how long should each cord be in order to show a beautiful formation after releasing them? (It should be noted that the pendulum wave is not considered a wave in physical terms, what we see is rather a joint view of independent pendulums.) You can see in the first color picture (Fig.1.) a snapshot of our pendulum wave. This phenomenon has many nice moments, perhaps the most beautiful is this front view.

Relatively little detailed literature related to the phenomenon can be found [1-3]. You can see videos of our [4] or other's [5] pendulum wave on the Internet. It is recommended to look at them before proceeding with the reading of this article. Two additional snapshots of our pendulum wave are shown in Fig.2. in top-side view. The parameters of the device are the same as the data in the later Table 1.

Its beauty and "obscurity" were the reasons why we would like to share our experiences with others. We hope that this beauty excites you, too.

THE PHYSICAL BACKGROUND

The physical background of the phenomenon is not difficult, but not obvious either. The "trick" is that the pendulums are adjusted in a way that the whole pendulum wave shall return to its starting position after certain time. Then all balls should be at the same position as their starting point again. During this time each pendulum swings with different frequency. For example, during the whole period of the pendulum wave the longest swings 52 times in total, the second swings 53, the next one 54 and so on. If we recognize this regularity, the problem isn't that difficult mathematically anymore.

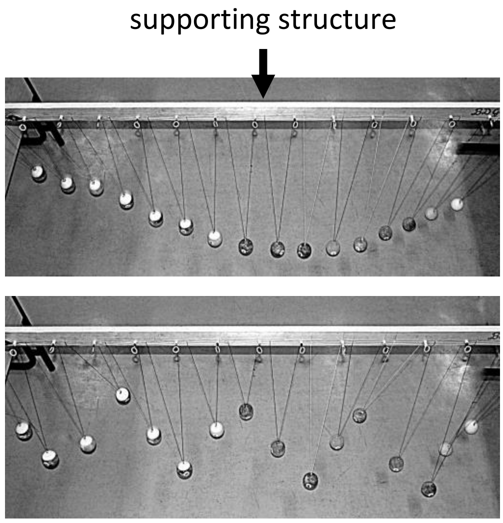

Fig.2. Snapshots of pendulum wave (top-side view)

We enumerate the pendulums: $i = 0,1,2,\dots n$. The pendulum wave consists of (n+1) balls. Pendulum No. 0 is the longest. We have additional symbols:

- $\boldsymbol{\tau}$ is the whole period of the pendulum wave (so the shortest time within which all pendulums in the pendulum wave return to the starting position).

- $\boldsymbol{T_i}$ is the period of the pendulum No. i (with the assumption of small amplitude swings).

- $\boldsymbol{N}$ is the number of swings made by the longest ($\mathrm{i} = 0$) pendulum during the time τ.

So the period of each pendulum is calculated according to this formula:

$$T_i = \frac{\tau}{N+i} \, . \qquad (1)$$

According to the well-known connection: $T = 2\pi \cdot \sqrt{\frac{\mathrm{l}}{\mathrm{g}}} \;\; \rightarrow \;\; l_i = \frac{g}{4\pi^2} \cdot {T_i}^2$, and from formula (1), with a given $\boldsymbol{\tau}$, $\boldsymbol{i}$ and $\boldsymbol{N}$, the length of each pendulum is found as:

$$l_i = \frac{g}{4\pi^2} \cdot \left(\frac{\tau}{N+i}\right)^2, \qquad (2)$$

where, of course, **g** is the gravitational acceleration. Thus, we have to choose the length of the rope No. $\boldsymbol{i}$ to be $\boldsymbol{l_i}$. This is the most important formula for us now.

Table 1. Our demo data

$\tau = 90\ s$	the whole period of the pendulum wave
$\mathrm{n} = 15$	$(\mathrm{n}+1) = 16$ pendulums
$\mathrm{N} = 52$	no. of swings made by the longest pendulum

CONSTRUCTION OF THE SET-UP

The following points are worth paying attention to during the building and the preparation of the set-up:

- a stable supporting structure (lath supported at both ends, for example by a table)
- the procurement of the balls (or other hanging objects)

- the selection of ropes (it shouldn't be breakable, or spinning), it's really hard to find good rope (the fishing line breaks, the embroidery yarn tears), we can find specialized shops on the Internet with the "twine, rope, cord, yarn" search words, then we must try them out with the swinging of only one ball
- an accurate suspension
- fine tuning and synchronization, this is the most important and the most difficult

To ensure this, set the lengths manually as accurately as possible, let the pendulum swing, and fine-tune the length of each pendulum by eye-measurement (extend or shorten). Small screws at the suspension of the pendulums are applied for this purpose. The tuning can be done with a computer method (for example: Webcam Laboratory Program), with that we are able to measure exact periods, but based on our experience, it isn't much better or easier. You should try it out, we don't have an exact recipe.

THE USE OF THE EQUIPMENT – THE INITIAL CONDITION

Figure 3. shows how we initiated our experimental runs. Initially, we displace the pendulums with a long, straight lath. However, based on our earlier calculations, the swings shouldn't be done with the same amplitude, but with the same starting angle for each. This does not make a large difference with the parameters we've used in practice, a line is a good approximation of the extremes' envelope. This we may not notice by just looking at videos, but can be found easily by watching a simulation based on exact mathematical formulae (as shown in the next paragraph). This means, that the lath should be long enough, to be able to start the system, but there is no real need of special equipment for this purpose.

Fig.3. The experiment at start (front view, data as in Table 1.)

COMPUTER SIMULATION

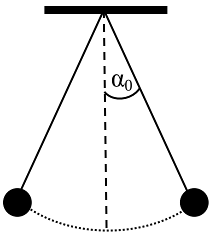

Fig.4.
The initial angle

As a further analysis of the pendulum wave, we implemented a simulation of it. To create an animation, we simply redraw the whole picture very frequently. To be able to draw it, we need to know every pendulum's length and its current angle at any moment. We already know the lengths (2). Assuming that the pendulums start without any initial velocity at time $t = 0$, the angle vs. time function is given as:

$$\alpha_i(\mathrm{t}) = \alpha_0 \cos\left(\sqrt{\frac{g}{l_i}}\,\mathrm{t}\right). \qquad (3)$$

Here, α_0 stands for the initial angle for all the pendulums (Fig.4). This formula (3) is an approximation, which uses the assumption that the value of $\alpha(t)$ is always small; this applies for all these pendulums. With these, we can calculate the coordinates of the whole system, so we can draw it.

Note that these approximations are considered to be really good with $\alpha_0 \leq 5°$, but that would not be spectacular enough. In practice (in the case of the experimental equipment and the simulation, too) we use angles more or less 10°, which is still good as an approximation, but there is a lot more movement, making it look better. The simulation allows more than 10° for the sake of spectacle, the program will continue to use the approximation.

Our program shows the front and top views of the model set-up (Fig.5. – it's a typical moment, which shows a similar shape for a prolonged period). The program has a feature which makes it possible to export data (rope lengths, etc.) with the current parameters, so that we don't have to make the calculations ourselves. The program is free-to-download as well [4] – directly download and run –, with the pre-requirement of having the Java Runtime Environment 7 installed [6].

The simulation can be stopped at any moment, or even put to an exact timestamp, as it is way more precise than the real experiment. By using the program, the formations are much easier to analyse. Of course, it can't substitute a real experiment.

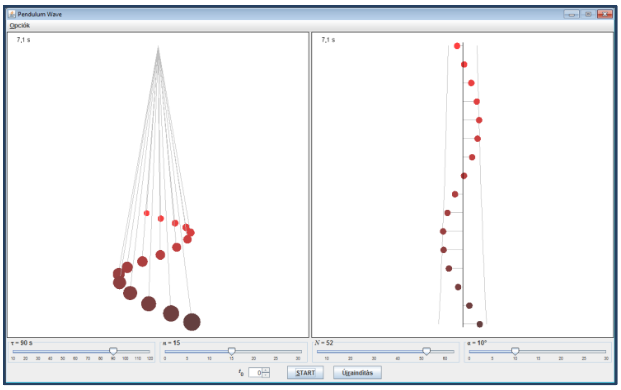

Fig.5. Simulation of a pendulum wave. Left: front view, right: top view ($t = 7.1\ s$, $\alpha_0 = 10°$), with the parameters of Table 1.

ANALYSIS

The moments we will analyse are the half and the one-third of the full period, in our case 45 s and 30 s (Table 2). The first column shows the pendulum's index, the second is the number of swings through the full period. The third and the fourth columns contain the number of swings done during these selected time intervals.

Table 2. Comparison of the number of swings within different time intervals: full (90 sec), half (45 sec) and one-third (30 sec) of period τ

Pendulum no.	**No. of swings ($\tau = 90\ s$)**	**No. of swings during 45 sec**	**No. of swings during 30 sec**
0	52	26	17 **1/3**
1	53	26 **1/2**	17 **2/3**
2	54	27	18
3	55	27 **1/2**	18 **1/3**
4	56	28	18 **2/3**
5	57	28 **1/2**	19
6	58	29	19 **1/3**
7	59	29 **1/2**	19 **2/3**
8	60	30	20
9	61	30 **1/2**	20 **1/3**
10	62	31	20 **2/3**
11	63	31 **1/2**	21
12	64	32	21 **1/3**
13	65	32 **1/2**	21 **2/3**
14	66	33	22
15	67	33 **1/2**	22 **1/3**

If we are at half time (45 s), the first pendulum did an integer number of swings, the next one did an integer and a half, then again an integer number, and so on. In perspective of positions, the number of fully completed swings does not matter, only the fractional part does. This means that every second pendulum is at the same position: the even ones are in their starting position, the odd ones are on the other side.

At one-third of the full period (30s), we can see in Table 2. that there are 3 different positions the pendulums can be in, with fractional parts of 1/3, 2/3 and 0. However, only two distinct positions are visible in Fig.6. (for convenience, the path of the two longest pendulums are also drawn be continuous arcs). This is because the 1/3 and 2/3 positions are the same, but the balls are moving in different directions. The 1/3's are still on the way to the opposite side, the 2/3's are coming back towards the starting position.

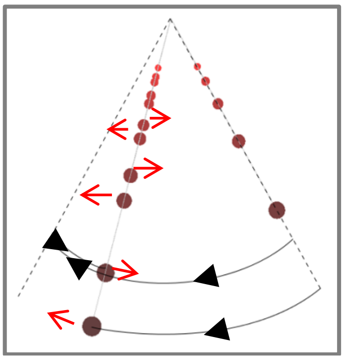

Fig.6. Pendulum positions (front view) at one-third of the full period (t = 30 s) in computer simulation with the parameter data as in Table 1.

Any moment can be analysed like these; we recommend considering the one-quarter (Fig.7.), one-sixth or two-fifths (Fig.8.) of the full period, as the constellations which occur in these moments are quite remarkable.

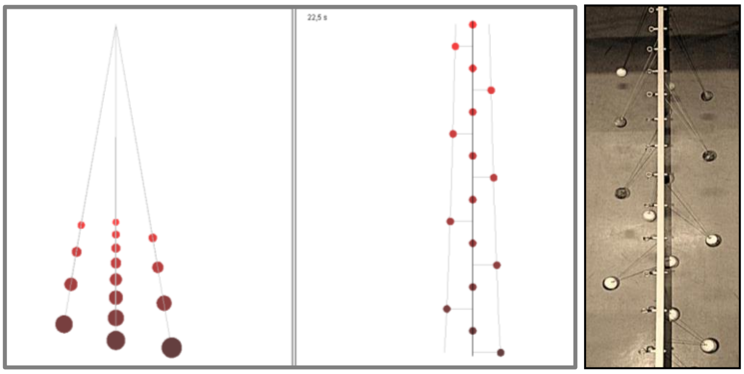

Fig.7. Positions at one-quarter of the full period
Left and middle panel: simulation, front and top views.
Right panel: photo of our set-up, top view, data as in Table 1.

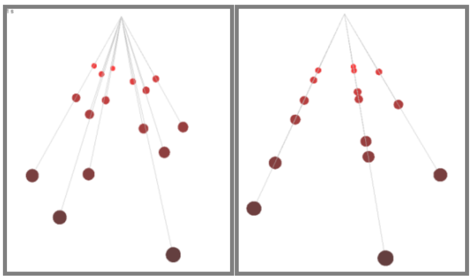

Fig.8. Nice shapes, left panel: 1/6 period, right panel: 2/5 period, simulation, front views, data as in Table 1.

CURIOSITIES

The simulation software did not learn physics: it does not know anything about Newton's laws, or gravitation, it only knows the formula, which is an approximation. Thanks to the missing knowledge of the program, we are able to see what this formula would give for angle values too large, where the small angle approximation does not hold. If we use $\alpha_0 = 120°$, the simulation is very spectacular, it makes shapes like butterflies. This sight can be observed really well on the move (for example in Fig.9. around half period). Even more beautiful, when the n-value is large. (You can see this in our simulation – "Enable great angles" button.)

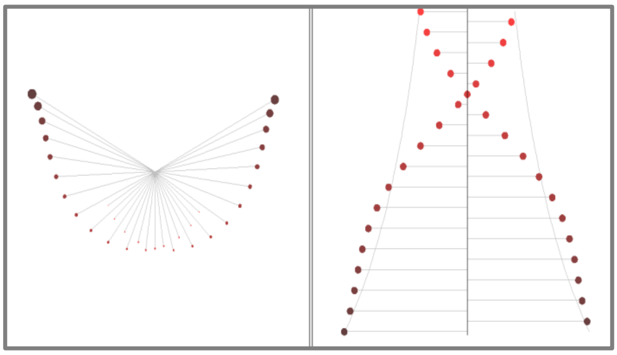

Fig.9. „Butterfly" → $\alpha_0 = 120°$,
Left: front view. Right: top view
$(n = 31; t = 44.1\ s; N = 52; \tau = 90\ s)$

We asked ourselves what would happen if the pendulum wave could "sing". Although building this in real life would be complicated, in the simulation we implemented this feature easily. The pendulums make sound in their leftmost and rightmost positions, and the longest pendulum gives the sound of a small „A", the next pendulum one semitone higher, and so on. The result was a beautiful, or – at least – quite interesting music. (You can hear this when running our program – "Enable sounds" button.)

We raised another question: what if we used a much simpler formula for the lengths, for example an arithmetic sequence. The beginning is very similar to the original one, it starts waving, but later, it turns into a system with no beautiful shapes at all. (You can see this in our program, too. – “Linear lengths” button.)

PHYSICS SCHOOL CAMP

Finally we would like to mention the physics school camp in our high school. Fig.10 shows two selected pictures of the camp. My school organizes a four-day physics camp each year, which forty-fifty students are attending from the school. In 2014, our pendulum wave-project was a great success.

The students have to work in smaller groups on a jointly chosen topic under the supervision of teachers. They present the projects to each other during the camp. The project’s framework might contain one or more experiments, measurements, evaluations, building of experimental equipment, preparation of computer simulation, theories or calculations. In the camp, the teachers also hold small group lessons. The programs are completed with invited speakers, team competitions, experiments and constant thought-provoking tasks. The location is usually an open-air school.

Fig.10. Top panel: observing of spectroscope, bottom panel: small group lesson with a physics program

OUTLOOK

You can find other versions of pendulum waves on the Internet [7]. For example a more spectacular version with fireballs, or one which is painted with fluorescent material, so it glows in the dark. We hope that some of you get a feel for preparing the equipment.

ACKNOWLEDGMENTS

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   Directly to the simulation: http://www.berzsenyi.hu/Lendvai/PendulumWave_eng.jar
5. Simple pendulum wave: https://www.youtube.com/watch?v=yVkdfJ9PkRQ
6. http://java.com/en/
7. Additional videos about other versions of pendulum waves:
   Pendulum wave with fireballs: https://www.youtube.com/watch?v=u00OF3ilNUs
   Pendulum wave in the dark: https://www.youtube.com/watch?v=7_AiV12XBbI
   Symmetrical pendulum wave: https://www.youtube.com/watch?v=vDtfWxL-AJg
   Spiral version with sound: https://www.youtube.com/watch?v=JMzB7sLeSbs