GAME THEORY IN SECONDARY SCHOOL

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ABSTRACT

Game theory gives a general mathematical framework to quantitatively describe real life situations and to characterize the interactions between players representing humans, groups of humans, animals, plants, bacteria, etc. The knowledge of the concepts and results of game theory can help us identify clearly the different types of interactions and to apply the known solutions in order to improve the life level of individuals and the economical efficiency of the societies. The terminology and the useful messages of game theory will be demonstrated by discussing a simple set of matrix games. Finally we summarize arguments justifying the importance of teaching game theory in the secondary school.

INTRODUCTION

Most of us use dozens of devices (e.g., mobile phone, computer, and LED lights) and enjoy the comfort provided by the modern buildings, cars, mass transportation, etc. All these things are the products of high technology utilizing the results of natural sciences including physics, chemistry, and mathematics. In the last century we have understood the structure of atoms, how materials are built up from atoms, and we can determine their macroscopic behaviour in the knowledge of microscopic interactions. At the same time, there are many other systems (human societies, ecological systems, biological species, languages, etc.) that are composed of many interacting objects.

The elementary interactions between the mentioned small objects are complex in comparison to those determining the behaviour of physical or chemical systems. In many cases, however, these interactions can be quantified by payoff matrices introduced in traditional game theory [1]. For example, in biological systems the strategies represent species and the payoffs quantify the effect of their interaction on their fitness measuring the capability of creating offspring. Evolutionary game theory [2], [3], [4] gives a general mathematical basis for investigating living systems and for understanding phenomena and mechanisms controlling human behaviour [5], [6], [7] and Darwinian evolution [8].

The application of the relevant results of game theory at different levels (from the individual behaviour to the decision of managers and law makers) is generally prevented by the absence of knowledge of the fundamental concepts of game theory and of the general nature of interactions described by games. Game theory quantifies a wide scale of situations we face day by day. Sometimes there exist many solutions (Nash equilibria) favouring different players. For example, in the coordination (or anti-coordination) type interactions the players benefit when choosing the same (or opposite) options whereas the difference in their payoffs may be the source of further conflict. For the so-called social dilemmas the individual interest suggests the players to choose options that are considered as “the tragedy of the community” because they could receive higher income with collaboration.

For the illustration of the terminology, concepts, solutions, and messages of game theory we first study the donation game and then a simple set of matrix games will be surveyed in the following sections. Finally we will discuss briefly experiences of teaching game theory in the secondary school and summarize additional arguments supporting the teaching of game theory.

DONATION GAME

Social dilemmas occur in a simple way in the donation game where two players (*X* and *Y*) have two options ("no" and "yes", in short *n* and *y*) to choose. More precisely, they must decide simultaneously (without communicating to each other) whether they wish to pay 1 euro to the co-player for receiving 2 euros. The pure incomes of players should be reduced by their investment as it is given in Table 1 for both players for all the four possibilities (called "strategy profiles" in game theory).

Table 1. Payoffs for player X and Y for the donation game.

X \ *Y*	No	yes
no	(0,0)	(2,-1)
yes	(-1,2)	(1,1)

The intelligent and rational (selfish) players recognize that their own investment is beneficial only for the co-player and the maximization of their own incomes advices both to choose "no". Here we have to emphasize that in traditional game theory the players assume that the co-players are also intelligent and rational players who wish to maximize their own payoffs irrespective of others. Thus the strategy profile (*n*,*n*) is a single Nash equilibrium that is the suggested solution in traditional game theory. In this case both players are satisfied because they cannot increase their own income by choosing another strategy unilaterally. At the same time the strategy profile (*y*,*y*) would provide them higher payoffs, and hence the dilemma.

The above Nash equilibrium can be found by determining the direction of edges in the flow graph for which the nodes represent strategy profiles and the edges connect those strategy profiles which differ in only one of the players' strategies as illustrated in Fig. 1. The directions of arrows point towards the strategy profile providing a higher income for the active player. In this directed graph the node without outgoing edges represents the Nash equilibrium.

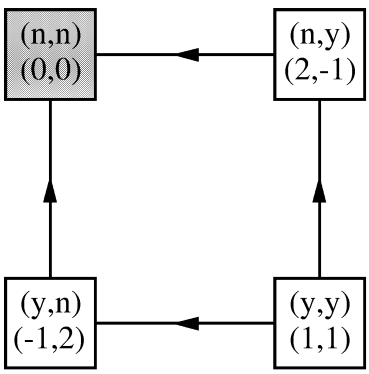

Fig.1. Flow graph of the donation game. The strategy profiles (nodes) are denoted by boxes indicating the strategy labels (upper rows) and payoff pairs (lower rows). The Nash equilibrium with only incoming edges is denoted by the gray box.

In real life situations the donation may be money, food, solicitude, transfer of knowledge or experience, and encouragement that has some cost *c* (here c=1) for the donator while the benefit b (here b=2) of donation exceeds the value of cost (in short, $b>c$). The essence of this game remains unchanged if the donations are not synchronized within a pair or when similar games are played by randomly selected pairs of a large population of players. At the same time, the repetition of these games implies a possibility to react to the previous decision of the co-player. In the latter (repeated or iterated) versions of these games the mutual donation can be enforced by following suitable strategies (e.g., tit-for-tat) [5]. Human experiments [6] have justified that in general people help each other in similar situations. Evolutionary game theory has explored several mechanisms supporting the maintenance of altruistic behaviour in similar situations [2], [3], [4].

SYMMETRIC MATRIX GAMES

In the above-mentioned donation game there are two equivalent players with both having two options what will be denoted henceforth as *D* and *C* strategies. Now we extend this 2x2 matrix game by introducing the bi-matrix notation. In this notation instead of the tabulated payoffs (see Table 1) the game *G* is defined by the following bi-matrices as

$$\mathrm{G} = \begin{pmatrix} (\mathrm{P},\mathrm{P}) & (\mathrm{T},\mathrm{S}) \\ (\mathrm{S},\mathrm{T}) & (\mathrm{R},\mathrm{R}) \end{pmatrix} \rightarrow \begin{pmatrix} (0,0) & (\mathrm{T},\mathrm{S}) \\ (\mathrm{S},\mathrm{T}) & (1,1) \end{pmatrix}, \qquad (1)$$

where the payoff pairs for each strategy pair are arranged in the same order as in Table 1. The equivalence of players for the symmetric games is reflected by the facts that the players receive the same payoff if both choose the same strategy and they exchange their payoffs when exchanging their strategies. As a result, all the symmetric 2×2 matrix games can be defined by four payoff parameters. The number of relevant parameters, however, can be reduced by identifying that the rank of payoffs (order of preference) remains unchanged if we add a constant payoff to each payoff component. Consequently, we can choose P=0 without any loss of generality. Additionally, we can modify the payoff unit by choosing R=1. The games with these rescaled payoff parameters are defined by only two parameters. Thus on the T-S plane each point represents a game. When evaluating the flow graph we can then distinguish only four types of the symmetric 2×2 matrix games separated by dashed-dotted lines in Fig.2.

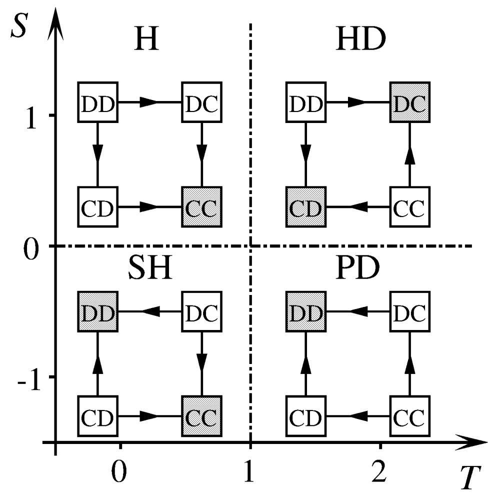

Fig.2. Flow graphs in the four quadrants of the T-S parameter plane where the gray boxes indicate Nash equilibria as in Fig. 1.

In the region of Harmony (H) games (T<1, S>0) the system has a single Nash equilibrium when the maxima of individual and common incomes coincide. In this notation the above-discussed donation game is located in the region (T>1, S<0) of Prisoner's Dilemma (PD) for which each game has a single Nash equilibrium. In the latter case, however, the choice of Nash equilibrium provides the same (zero) income for the players that is less than what they would get when choosing the opposite strategy mutually. Unfortunately, the original story of the prisoner's dilemma [9] masks and suppresses the importance of this serious social dilemma we face frequently day by day. For example, such situation can occur in the recent level of specialization (division of work) when the workers decide whether they execute their duty fairly or not. Similar situations can be observed among bacteria having two mutants (considered as strategies) producing (or not) enzymes that catalyze the extraction of food from the environment [10]. The disarmament negotiations and many other examples of social (or prisoner's) dilemmas indicate clearly that the exploitation of others is not a human invention rather it should be considered as an existing type of interactions between living objects.

The Stag Hunt (SH) game represents an interaction where the players do the best when choosing the same options, consequently, this game has two Nash equilibria. At the same time their income may depend on the strategy they choose mutually and the difference in their payoff may be the source of an additional conflict. Examples for these real life situations are cases when the players have two options (to use meter or inch as length unit, to install Linux or Windows on their computers, to select one of the possible technologies in their work, etc.) and some advantage can be realized when choosing the same one. This is the reason why these games are frequently termed as coordination games.

Using the latter terminology the Hawk-Dove (HD) game is an anti-coordination game because here the system has two Nash equilibria when the players should choose the opposite options (see Fig. 2). Such situations occur also frequently in the mentioned systems. In biological systems the symbiotic behaviour exemplifies this type of interactions. In human societies the division of work (or specialization) can create extra profit for the players if they divide the task(s) into complementary parts. Anyway, this is the game that two players play when entering a room through a narrow door as both player have two options: to be the first or second to enter.

Fig.1. illustrates that we can distinguish only four types of symmetric two-player two-strategy games whereas we can draw 2^4=16 different flow graphs for the non-symmetric games. The reduction in the number of distinct types is related to the symmetry ensuring the same payoff variation for both players when deviating unilaterally from the DD or CC strategy profiles. The latter symmetry implies an additional consequence, namely, the sum of the payoff variations (of the active players) is zero along the four-edge loop of this flow graph. On the analogy of the potential energy in physical systems here we can also introduce a potential for each strategy profile that can be given by the potential matrix $\boldsymbol{V}$. In the present example the payoff matrix $\boldsymbol{A}$ and the potential matrix $\boldsymbol{V}$ are expressed as

$$A = \begin{pmatrix} 0 & T \\ S & 1 \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} 0 & S \\ S & 1-T+S \end{pmatrix}, \tag{2}$$

where the matrix components, defining payoffs for the first player and potentials for the pair of players for all the strategy profiles, are arranged in the same way as in Table 1 and Eq. (1). In fact, the potential summarizes the individual incentives along a series of changes in the space of strategy profiles when only one player can modify her strategy. The latter property is reflected by the equivalent payoff differences in the corresponding columns of $\boldsymbol{A}$ and $\boldsymbol{V}$. Notice that $\boldsymbol{V}$ is a symmetric matrix. The largest component of the potential matrix plays a distinguished role as it identifies a preferred Nash equilibrium. More precisely, the corresponding strategy profile is analogous to the ground state (minimizing the potential energy) in physical systems. The left panel of Fig. 3 indicates the preferred Nash equilibria as a function of the payoff parameters. The mentioned preference selects one of the Nash equilibria in the region of SH game whereas the equivalence of the DC and CD Nash equilibria is reserved for the HD games. It is emphasized that the state of the "tragedy of the community" (gray territory in Fig. 3) emerges within a large region of parameters.

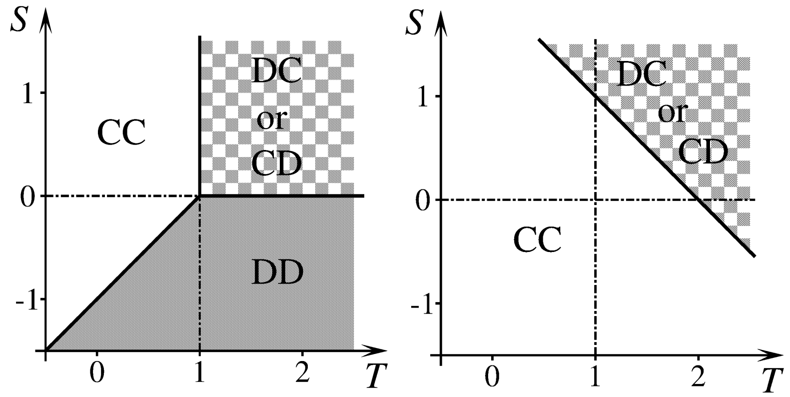

Fig.3. Left panel: Preferred Nash equilibria for selfish (rational) players on the T-S plane.
Right panel: Nash equilibria for fraternal players for the same payoffs.

The right panel of Fig. 3 illustrates a way how the "tragedy of the community" can be avoided by fraternal players who represent the behaviour of family members or friends. The fraternal players agree to share their total payoffs equally. In that case the individual and common interests coincide and the dilemma vanishes.

Two Nash equilibria exist in the regions decorated by checkerboard patterns in Fig.3. In these solutions the players receive different payoffs, namely, T and S. The latter differences, however, can be eliminated for iterated games if the players alternate the strategy profiles CD and DC.

In the multi-agent spatial evolutionary games [11] the players are distributed on a lattice and they play games with their neighbours. Sometimes the players are allowed to modify their strategy by following a dynamical rule. In contrary to the traditional game theory [1] here the intelligence of the equivalent players is limited. In biological models the unsuccessful players are replaced by the offspring of the more successful ones in the spirit of Darwinian selection [2]. This approach is adapted for human systems with assuming that players with low payoffs imitate/adopt the strategy of a more successful neighbouring co-player. The so-called logit rule assumes a higher level of intelligence, because here the players are capable of evaluating their payoffs for their possible strategies at a fixed strategy profile in the neighbourhood. In consecutive elementary steps they choose one of their strategies with a probability increasing exponentially with the payoff. This stochastic rule drives the systems into the Boltzmann distribution that validates the concepts and mathematical tools of statistical and solid state physics [12], [13], [14].

Fig.4. illustrates a process that exemplifies the close relationship (analogy) between the physical and social systems (for a more detailed discussion of this analogy see [15]). The physical system is the ferromagnetic Ising model where atoms are located on the sites of a square lattice and the magnetic moments of atoms can be oriented upward (↑) or downward (↓). For the ferromagnetic materials the interactions between the neighbouring atoms favours the same orientation, namely, ↑↑ or ↓↓. At low temperatures we can observe and ordered state when all the magnetic moments point to the same direction. In the presence of an external

magnetic field, however, one of these ordered states is preferred. Fig.4. show a reversal of magnetization when initially most of the magnetic moments point to downward despite of the presence of an external magnetic field preferring the opposite ordered arrangement. Due to the stochastic events atoms with ↑ state occur occasionally but their moments are reversed back by the neighbouring atoms within a short time. Sometimes, however, an island of ↑ atoms can be formed, and if its size is sufficiently large then the size of this island grows with a velocity proportional to the strength of the external magnetic field. The related phenomena are well described in the literature of magnetic materials and this knowledge is utilized in magnetic hard discs, electric motors, and transformers.

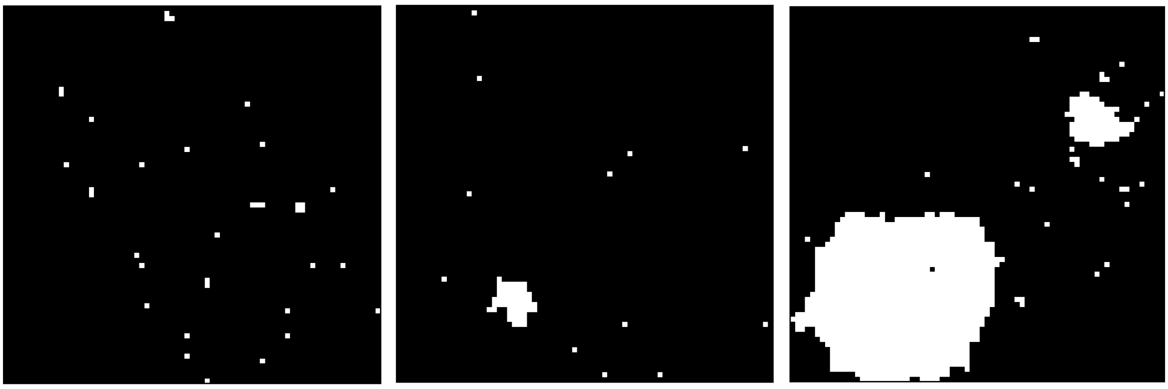

Fig.4. Evolution of strategy distribution for the evolutionary stag hunt game on a square lattice for S=-0.4 and T=0.5 at a low noise level. Initially almost all the players use D strategy indicated by black boxes on the left snapshot. The system remains in this state until the formation of a sufficiently large colony of C players denoted by white boxes in the middle snapshot. The territories of these C islands growth and finally (right snapshot) the system evolves into the preferred Nash equilibrium where some defects can be present due to mistakes in the imitation.

In social systems the above process can be interpreted, for example, as the spreading of a new technology in a society where initially the members (players) use an older and less efficient one. In these models the strength of noise quantifies the frequency of mistakes in the strategy refreshments and its role is analogous to the temperature in physical systems.

Besides the above examples there are many other potential games even with more than two strategies that can be mapped onto a suitable physical model [12], [13]. It is known that the presence of cyclic dominance in the payoffs prevents the existence of potential [14].

The cyclic dominance is represented by the matching pennies and rock-paper-scissors game for two or three strategies [2], [3]. The presence of these components in the interactions can help the maintenance of cooperative behaviour in the social dilemmas. Anyway, the games with cyclic dominance exhibit some other surprising and general laws (e.g., unexpected consequence of external influence [16] and parity effects [17]) that are also worthy of teaching.

MESSAGES OF GAME THEORY

The above examples have illustrated how game theory can be used for the quantitative explanation and classification of phenomena affecting our life directly. Now we underline several properties that are not detailed previously.

First we emphasize the difference between game theory and decision theory that can be considered as the first lecture in a course of game theory. Decision theory deals with cases when we should select one of the possible options whose utilities are quantified by real numbers (the better choice is quantified by a larger payoff). Here we can recognize that the valuation of utilities may depend on the player. Furthermore, the determination of the optimal choice is based on the rank of payoffs that simplifies the quantification of utilities.

In non-cooperative game theory we study situations where n players choose one their possible options simultaneously (without collaboration or communication with any of the others) and the quantified utilities depend on the choices of all the participants. For most of these normal games the players cannot choose the strategy providing their highest individual income because of the counter-interest of the co-players. The traditional game theory assumes rational and intelligent players who wish to maximize their own payoffs (irrespectively of what happens to the others) meanwhile they also assume the same levels of selfishness and intelligence for the co-players. For normal games the Nash theorem [18] states the existence of (at least one) so-called Nash equilibria from which the unilateral deviation is not beneficial for the players. In many games (e.g., matching pennies or rock-paper-scissors) only one mixed Nash equilibrium exists when the players choose one of their pure strategies with probabilities depending on the payoffs. In these cases the game resembles gambling.

Fundamentally different difficulties arise for games having two or more Nash equilibria because here we need further criteria to choose one of them. The most important problems, however, are related to the social dilemmas that exist in a large region of the payoff parameters for the normal games. As mentioned above, in social dilemmas the individual selfishness enforces the players to choose a Nash equilibrium that may provide a low income in comparison to those suggested for their collaboration. At the same time for the latter strategy profile(s) some of the players can benefit when deviating unilaterally from the collaboration and these possibilities destabilize the collaboration.

In the light of game theory the mentioned features can be considered as natural laws characterizing the interactions between living objects. The systematic investigations of the models of evolutionary game theory have already explored relevant effects and consequences of these types of interactions [3], [4], [11]. Different mechanisms are discovered that can help the society to avoid the trap of social dilemmas. For example, the application of the tit-for-tat (TFT) strategies was suggested by Axelrod [5] for repeated prisoner’s dilemma game at fixed partnership in the absence of noise. The TFT strategy algorithm chooses C strategy in the first step, afterwards the TFT players repeat the co-player's previous step that can be interpreted as a punishment of defection and reward of cooperation. In the presence of noise (mistakes), however, the use of the forgiving TFT strategy is more beneficial [3], [4], [11]. Nowak and Sigmund [3], [4] have evaluated the probability of forgiving (when the defection is not reciprocated) that depends on the payoffs. In the absence of forgiveness the strict TFT players would punish alternately each other after an occasional mistake.

Other possibilities [e.g., punishment, reward, and voluntarism] are also explored and can be efficiently used in repeated human interactions. Some of the mentioned methods require additional cost to be paid by the players favoring the maintenance of cooperative behavior that is beneficial for the whole community. This is the reason why the altruistic punishment is considered as a second order donation game.

The limited length of this article does not allow us to discuss other general laws and relationships that can be analysed with using the concepts of game theory. Instead of it in the following section we briefly survey other arguments supporting the importance of teaching game theory in high schools.

ADDITIONAL ADVANTAGES AND CHALLENGES IN TEACHING GAME THEORY

Children enjoy games. In the ancient ages as well as today games have helped us develop skills in order to find a good solution quickly in every-day situations. Theachers [15], [19], have reported significant increase in the activity of students in courses of game theory. A relevant portion of students have modied their opinion of learning math and other natural sciences as they found the related problems attractive and useful. In other words, the teaching of game theory has generally improved the reputation of natural sciences including the approaches and methods of physics.

Computers and internet ensure an excellent background/facility for everyone to play the games of game theory against each other. In the last years Alvin Roth (Nobel Memorial Prize Laurate in Economics, 2012) and his team has launched a softver that is already used in several universities in teaching game theory [20]. Teachers and students can register themselves and play games under the conduction of their teacher. When playing repeated games against each other or against the computer the students can experience personally how to use mixed strategies in the matching pennies or rock-paper-scissors games.

For the repeated social dilemmas the players can do a series of experiments in order to find good solutions against opponents following unconditional cooperation or defection, different versions of TFT, random, or win-stay-lose-shift strategies. The necessity of forgiveness between to TFT players can also be well demonstrated in situations where mistakes are present.

The investigation of social dilemmas cannot be separated from ethical questions because the unconditional cooperators and defectors can be considered as good and bad members of the community. The mentioned games illustrates the usefulness of the fraternal behavior. Here it is worth mentioning that the importance of altruism and trust in human behavior are also investigated experimentally and the results are justified by mathematical models of evolutionary game theory.

For example, in the two-player ultimatum game [21] the first player proposes how to share 100 dollars between them and the second player can accept or reject it. If the proposal is accepted then both players recieve the proposed portions, otherwise the players receive nothing. In this game the Nash equilibrium is to propose only 1 dollar for the co-player who should receive it as it is more than nothing. On the contrary, the human experiments indicated that a large portion of players suggested approximately equal sharing. The latter result is related strongly to the experimental fact that most of the players rejected the proposal less than 20 dollars. These experiments are performed by students who know nothing on game theory and social dilemmas. Otherwise the preliminary knowledges modify the behavior of players [22]. In other words, the acceptance of social norms (here the fair sharing of income) can be trained by playing similar games.

It is worth mentioning that the necessary trust in human collaborations can also be improved by playing the so-called trust game [23]. In this experimental game the players are located in two rooms (A and B) separately. Players in room A have to decide how much of their 10 dollars is invested and sent to a counter-player (in room B) who receives a tripled sum. Afterward players in room B have to decide how much of the tripled money to keep and how much to send back to their respective coplayer. The experiments indicated a wide scale in the behavior that is improved when the game is repeated by the same players later.

In traditional game theory cooperative games are addressed to study the formation of strategy associations that can provide higher total income for the members of collaborators. In evolutionary game theory [10], [11] dozens of models have justified the spontaneous emergence of strategy associations via the Darwinian selection. For example, in sports the success of the specialization of players in a football or handball team is well demonstrated. Additionally the latter examples illustrate both the importance of high level training (teaching) and of finding the best persons for all positions.

Finally we have to mention briefly the challenges and difficulties coming from the absence of traditions and methodology in teaching game theory. Playing different types of games can be adjusted to the evolution of the human brain. The experimental investigations of children's behaviour [24], [25] have clearly indicated that human egalitarianism and parochialism have deep developmental roots, the inequality aversion and other-regarding preferences develop (and can be trained) strongly between the ages of 3 and 8. The mathematical knowledge of high school students makes them capable of analysing the above-mentioned problems and phenomena with the help of game theory and also discovering the similarities between different living systems. The above mentioned examples have demonstrated the application of some simple mathematical concepts and methods that can motivate students to learn more on vectors, matrices, and graphs. At the same time it implies the necessity of collaborations between teaching mathematics, physics, chemistry, biology, and ethics.

CONCLUSIONS

Game theory originally was developed to find the solutions in some traditional games (like poker and other card games, nim game, table games, military games, etc.) with using the tools of mathematics. It turned out, however, that the concepts of game theory can be efficiently used to study the general features of phisical and living systems where the complex interactions are characterized by payoff matrices representing social dilemmas, cyclic or hierarchical dominance between the distinguishable players. Modern game theory and evolutionary game theory give us a tool for explaining analogous phenomena occuring in different fields of sciences. All these advantages can be utilized if students learn the alphabet of game theory at the level of their knowlegde of algebra and geometry.

The efficiency of teaching game theory can be increased significantly by applying the tools of informatics as detailed above. These facilities can maintain and enhance the interest and activity of students int he high schools and will help them to improve the level of cooperation/collaboration in their future life.

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