Transformations in physics

Teaching Physics Innovatively

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The aim of lecture

to show interesting examples of the use of transformations

- The first example: the utility of the use an appropriate coordinate system in the classical physics (reference system of centre of mass) – see literature in [1]

- The second example: the role of Lorentz-transformation at understanding of wavefunction in quantum mechanics

The colliding bodies and the centre of mass

Data for calculation:

$m_1 = 3$ kg
$v_1 = 4$ m/s

$m_2 = 2$ kg
$v_2 = -9$ m/s

The centre of mass moves vith a constant velocity.

The system of quadratic equations

- In calculations we use laws of conservation of energy and momentum, and we solve the system of equations:

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$\tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 = \tfrac{1}{2} m_1 u_1^2 + \tfrac{1}{2} m_2 u_2^2$$

u_1 and u_2 are the velocities after collision

- The result: $u_1 = -6{,}4$ m/s and $u_2 = 6{,}6$ m/s

The transformation to the system of centre of mass

- The velocity ***V*** of the centre of mass:

$V = (m_1 v_1 + m_2 v_2) / (m_1 + m_2)$ **$V = -1{,}2$ m/s**

- $v_1 \Rightarrow v_1 - V$: 4 m/s $\Rightarrow$ 5,2 m/s

$v_2 \Rightarrow v_2 - V$: – 9 m/s $\Rightarrow$ – 7,8 m/s

- The change of signs: 5,2 m/s → – 5,2 m/s

-7,8 m/s → 7,8 m/s

- The result: $u_1 = -5{,}2 \text{ m/s} + V = -6{,}4$ **m/s**

$u_2 = 7{,}8 \text{ m/s} + V = 6{,}6$ **m/s**

The Lorentz-transformation

- The system *K'* moves parallel to the x-axis of system *K* at a constant speed *v*

- The conventional relationships:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

(a) (b)

- Because both *x'* and *t'* depends on *x* and *t*, it is suitable to transform events from system *K* to *K'*:

$(x, t) \longrightarrow (x', t')$

The wavefunction

- The wavefunction plays a central rule in quantum-mechanics, it is a mathematical formulation of physical properties of matter-waves

- Each quantum-mechanical textbook use the formula

$$\lambda = h / (mv)$$

which gives the wavelength λ of a quantum with mass m and velocity v, h is a Planck constant.

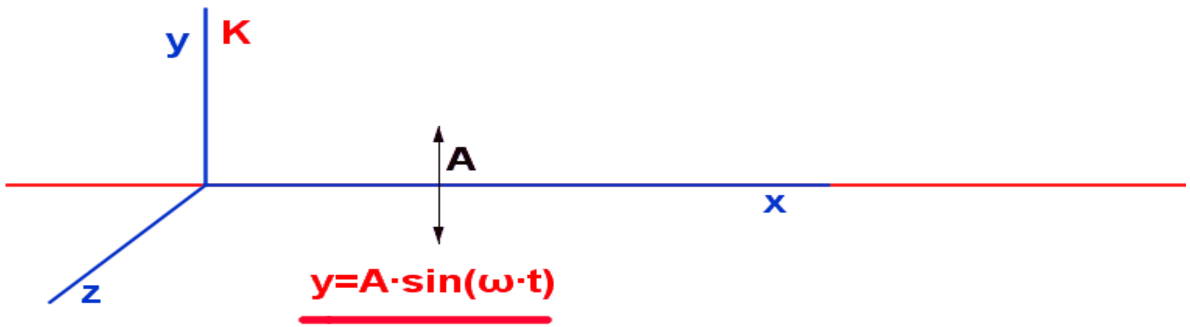
What have to do Lorentz-transformation with wavefunction?

- Standing waves with unlimited wavelength and frequency $m_0 c^2 / h$ are converted into de Broglie waves by Lorentz transformation.

- In the literature we see less elaborated and other ways of derivations, but with more voluminous discussions, e.g. in [2, 3, 4, 5].

What we transform?

- The function $y = A \cdot \sin(\omega \cdot t)$ gives values *y* accordingly to the reference frame *K*. In *K* we have standing waves of unlimited wavelength.



- Then we consider values of this function in a system *K'* which moves parallel to the x-axis at a constant speed *v*.

How we transform?

- We need $t' = const$, that is $t - v \cdot x / c^2 = const$ accordingly relation (b) of Lorentz-transformation.

- If the condition $T = v \cdot \Delta x / c^2$, otherwise $\Delta x = T c^2 / v$ is valid, the pair of events of data (x, t) and $(x + \Delta x, t + T)$ transforms in a simultaneous pair of events in frame K'.

The transformation

- We transform the pair of events of data (x, t) and ($x + Tc^2/V$, $t + T$) into frame K'.

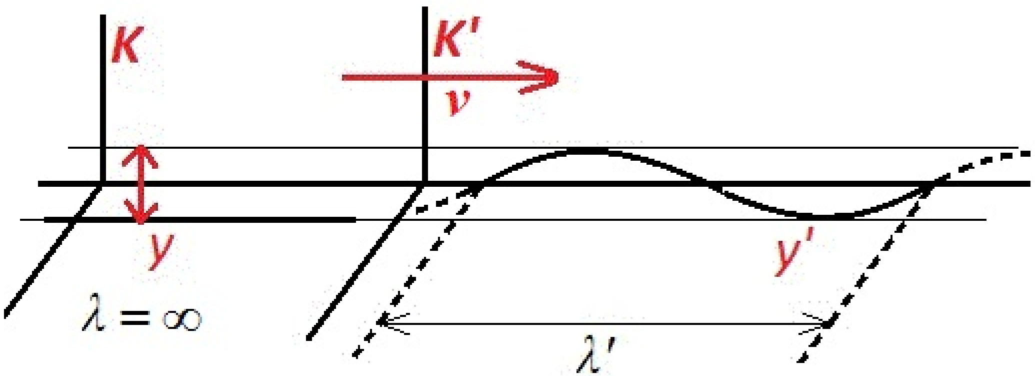
- The events are transformed to the coordinate

$$x_1' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad x_2' = \frac{x + \frac{Tc^2}{v} - v(t+T)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- their distance is the sought wavelength λ'.

The result of transformation

- $\lambda' = x_2' - x_1' = \dfrac{\dfrac{Tc^2}{v} - vT}{\sqrt{1 - \dfrac{v^2}{c^2}}}$ $\quad \lambda' = T\sqrt{1 - \dfrac{v^2}{c^2}} \cdot \left(\dfrac{c^2}{v}\right)$



what we see from frame K ***what we see from frame K'***

- A standing wave of infinite wavelength in frame *K* transforms to the propagating wave of wavelength λ' and phase velocity c^2 / v in frame *K'*.

The de Broglie waves

- Let the frequency in the frame *K* is the rest frequency $m_0 c^2/h$ of a quantum.

Then $\lambda' = \dfrac{h}{m_0 c^2}\sqrt{1-\dfrac{v^2}{c^2}}\left(\dfrac{c^2}{v}\right)$ and

if we consider $m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$ $\lambda' = h / (mv)$.

Thus we have de Broglie waves in the frame *K'* in a general, relativistic case.

Further relations

- We see from the formula $\lambda' = T\sqrt{1-\frac{v^2}{c^2}}\cdot\left(\frac{c^2}{v}\right)$

$$\lambda' = T' \cdot c^2 / v = T' \cdot v_f$$

where $v_f = c^2 / v$ is the phase velocity of de Broglie waves. In the case $v < c$ the phase velocity of matter waves exceeds *c*.

- The velocity *v* of frame *K'* can be considered as the velocity of the quantum – group velocity v_g .
- We see also $v_f \cdot v_g = c^2$

Is the quantum point-like?

- We can see from the presented derivation that we get de Broglie wave only by transforming a three-dimensional standing wave.
- Hence the derivation also points out that the quantum cannot be point-like.
- See additional disscusion in [6, 7, 8].

Literature

1. L.D.Landau-E.M.Lifsic:Elméleti fizika I. Mechanika. Tankönyvkiadó Budapest, 1974.

2. J.M. Espinoza: Physical properties of de Broglies phase waves. A. J. Phys. , Vol 50, No. 4. April 1982.

3. J. W. G. Wignall:De Broglie Waves and the Nature of Mass. Foundations of Physics, Vol. 15, No. 2, 1985.

4. Edward MacKinnon:De Broglie’s thesis: A critical retrospective. Am. J. Phys. 44, 1047 (1976).

5. Harvey R. Brown and Roberto de A. Martins:De Broglie’s relativistic phase waves and wave groups. Am. J. Phys. 52, 1130 (1984).

Further literature from the Internet

6. https://en.wikipedia.org/wiki/Point_particle

7. http://www.massline.org/Philosophy/ScottH/infinitely_small.htm#n6

8. http://physics.stackexchange.com/questions/41676/why-do-physicists-believe-that-particles-are-pointlike

Thank You for Your Attention.